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Simple Technique of Projected Lagrange for a Class of Multi-Stage Stochastic Nonlinear Programs

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Abstract

This paper presents a techniq for solving multi-stage stochastic nonlinear programs. The techniq is based on projected lagrange approach which generates the search direction by solving parallelly a set of quadratic programming subproblems with size much less than the original problem at each iteration. Mathamatically, can be pointed out that Lagrange's projection method can solve problem multi-stage stochastic nonlinear programs.

Keywords: Multi-stage stochastic nonlinear programs; Projected langrange; Scenario analysis; Decomposition

Introduction

Multi-stage stochastic nonlinear programs emerges in there are many practical situation, as production and manpower planning, portfolio's selection etc.. Have a lot of research that gets contribution to solve Multi-stage stochastic nonlinear programs amongst those methodics decomposition that is utilized on nonliner's program also linear [1,2]. Most of that literature in reference to principle decomposition that introduced by Dantzig and Wide [3].

Decomposition method experience development which is with marks sense L-shape decomposition method that enough effective being utilized to troubleshoot multi-stage stochastic nonlinear programs

Severally methods the other to troubleshoot multi-stage stochastic nonlinear programs is analyzed among those by Birge [4]. Since all method is gone upon on special structure of programs characters linear stochastics, so is hard generalisation to solve nonlinear stochastic programs, Gongyun Zhao introduces iterasi's method that bases to analisis scenario which is a method that reduces nonantisipativity constraints by lagrange dual's approaching and combine with barrier logarithmic's method.

Hereafter Gongyun Zhao [5] developing decomposition method which is by use of sequential quadratic method's programming. Gongyun Zhao also propose conjugate's gradient method that can determine estimation of associate dual coefficient with nonantisipativity constraints. Gongyun Zhao [5] also develop lagrangian dual's method to solve nonlinear's program multi's stochastic phase.

Lagrange's projection algorithm was analyzed by Murtagh and Sander [6] to troubleshoot sparse nonlinear constraints. Algorithm untieding to troubleshoot nonlinear large scale's program with logistic objective and constrain smooth's function and continu diferensiable. Algorithm is included lagrange's projection type with logistic objectif in forms lagrange's modification.

Base research already being done researchers former to solve multi-stage stochastic nonlinear program, in this paper, writer propose a method for solve to program multi-stage stochastic nonlinear programs which is by use of lagrange's projection method. This method is expected gets to give alternative solution to solve multi-stage stochastic nonlinear programs.

Materials and Methods

Consider the following multi-stage stochastic program with recourse:

$$\min_{\chi\in X\hat{C}_0}(x)+E_{\xi_1}Q_1(x_1\xi_1) \tag{1.1}$$

Where $X=\{x|c_0(x)=0\}\subseteq\mathfrak{R}^{n_0}$,

the recourse function

$$Q_1(x,\hat{\xi}_1)=\min_{y_1}q_1(x,y_1,\xi_1)+E_{\xi_2}Q_2(x,y_1,\hat{\xi}_1,\xi_2)$$

$$\text{Subject to } c_1(x,y_1,\xi_1)=0 \tag{1.2}$$

And for $t=2,...,T-1$, recursively we have

$$Q_t(x,y_1,..,y_{t-1},\hat{\xi}_1,..,\xi_t)=\min_{y_t}q_t(x,y_1,..,y_{t-1},\hat{\xi}_1,..,\hat{\xi}_t)+E_{\xi_{t+1}}Q_{t+1}(x,y_1,..,y_t,\hat{\xi}_1,..,\hat{\xi}_t,\xi_{t+1}) \tag{1.3}$$

$$\text{Subject to } c_t(x,y_1,..,y_t,\hat{\xi}_1,..,\hat{\xi}_t)=0 \tag{1.4}$$

$Q_T=0$. $x\in\mathfrak{R}^{n_0}$ is the deterministic vector, ξ_i is the realization of the random vector ξ_i. $y_i\in\mathfrak{R}^{n_i}$ is the decision vector in the i-th stage, which is generated recursively by $x,y_1,..,y_{i-1}$ and $\hat{\xi}_1,..,\hat{\xi}_i$, hence represents $y_i(x,y_1,..,y_{i-1},\hat{\xi}_1,..,\hat{\xi}_i)$ actually. $\hat{c}_0$ and c_0 are real-valued functions on $\mathfrak{R}^{n_0}$ .c_t is random since it is related to $\hat{\xi}_1,..,\hat{\xi}_t$

For the discrete random vector $\xi=(\xi_1,..,\xi_{T-1})$ if c_t has finite realizations $c_{it}(i=1,..,S_t)$, then all these c_{ti} form the constraint functions on stage *t*. The details on the formulation of multi-stage stochastic programs can be found, e.g. in Kall and Wallace [7].

Let $\xi=(\xi_1,..,\xi_{T-1})$ and assume that (Ω,θ,P) is the associated probability space. Suppose that we have S scenarios $\xi^{(s)}=(\xi_1^{(s)},\xi_2^{(s)},..,\xi_{T-1}^{(s)})$, which has a fixed and known probability distribution $\{(\xi^{(s)},p_s|s=1,2,..,S\}$. Then (1.1)-(1.4) can be reformulated as the following nonlinear programming problem:

$$\min\sum_{n=1}^{S}f_s(z^{(s)}) \tag{1.5}$$

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$$s.t.h_s(z^{(s)}) = o, s = 1,2,..S \quad (1.6)$$

$$\sum_{s=1}^{S} A_s(z^{(s)}) = 0 \quad (1.7)$$

where $z^{(s)} = (x^{(s)}, y_1^{(s)}, .., y_{T-1}^{(s)} \in \Re^n, n = \sum_{i=0}^{T-1} n_i$

$$f_s(z^{(s)}) = p_s(\hat{c}_0(x^{(s)}) + \sum_{t=1}^{T-1} q_t(x^{(s)}, y_1^{(s)}, .., y_t^{(s)}, \xi_1^{(s)}, .., \xi_t^{(s)})$$

$$h_s(z^{(s)}) = (c_0(x^{(s)}); c_1(x^{(s)}, y_1^{(s)}, \xi_1^{(s)}); .., c_{T-1}(x^{(s)}, y_1^{(s)}, .., y_{T-1}^{(s)}, \xi_1^{(s)}, .., \xi_{T-1}^{(s)}))$$

Constraints (1.7) are the so-called non-anticipativity constraints, which reflect the fact that scenarios sharing a common history up to any moment of time must have a common decision up to that moment. Readers can refer to Rockafellar and Wets [8] for more details on this reformulation.

Lagrange's function in common is deep shaped as follows,

$$L(x,q) = f(x) + q^T Ax$$

For q parameter.

Let say to be given by parameters *q* and *r*, therefore form augmented lagrange of equation upon is as follows,

$L(x,q,r) = f(x) + q^T Ax + \frac{1}{2} r \|Ax\|^2$ Let say to be given x_k, μ_k, ρ, so,

$$\min L(x, x_k, \mu_k \rho),$$

$$s.t.\tilde{f} = 0, 1 \le x \le u$$

To a $\overline{f} = Ax$.

Therefore objective function for problem to multi-stage stochastic nonlinear programs is as follows,

$$L(x, x_k, \mu_k, \rho) = f(x) + \mu_k^T (f - \tilde{f})$$

Result and Discussion

In this paper, Lagrange's projection method will be utilized for multi-stage stochastic nonlinear programs. The methods based lagrange augmented modified.

Assume that $f(s): \Re^n \to \Re$ and $h(s): \Re^n \to \Re^{m_s}$ are two continu diferensiable functions, $h_s: \Re^n \to \Re(i = 1, \cdots m_s)$ and $h_s(z^{(s)}) = (h_{s1}(z^{(s)}) \cdots, h_{m_s}(z^{(s)})^T, A_i \in \Re^{m_0 \times n}, A = (A_1, \cdots, A_s) \in \Re^{m_0 \times \delta}$ is matrix row with full rank and has special structure.

So equation (1.5) – (1.7) are formulated as forms as follows, *min* f(x)

$$s.t.h(x) = 0, 1 \le x \le u \;\; s.t.h(x) = 0, 1 \le x \le u$$

$$A(x) = 0$$

Where are matrix m x n with $m \le n$.

And with $f(x) = \sum_{s=1}^{S} f_s(z^{(s)})$ $h(x) = h_s z^{(s)}$ and $A(x) = A_s z^{(s)}$

That note function of x assumed continu diferensiable.

With Lagrange's projection method, objektif's function takings *f(x)* one equal to form commons of Lagrange augmented's functions,

$$f(x) = -\lambda^T A(x) + \frac{\rho}{2} A(x) A^T(x)$$

Vector λ^T is lagrange's coefficient vector and ρ is penalti's parameter.

Linear Aproksimation from constraints nonlinear is make iterasi along starting point *x(k)* from iterasi process followings;

$$A(x(k+1)) = A(x(k)) + h(k)(x(k+1) - x(k))$$

So algorithm that presented to solve subproblem constrain's line linear with function objective is modify lagrange augmented and linear aproksimation *f(x)* on the *x(k)* are as follows

$$\min \mathcal{L}(x, x_k, \lambda_k, \rho) \quad (1.8)$$

$$s.t\overline{A} = 0, 1 \le x \le u$$

Where is function objective is modify lagrange augmented and $\overline{A}$ is aproksimasi $f(x)$ on the *x(k)* and,

$$\mathcal{L}(x, x_k, \lambda_k, \rho) = f(x) - \lambda_k^T (f - \overline{f}) + \frac{\rho}{2} (f - \overline{f})^T (f - \overline{f})$$

$$\overline{f} = f_k + J_k(x - x_k)$$

Where f_k and J_k is cconstraints vector and jacobi's matrix that evaluated with *x(k)*.

Definition 1. To $\rho = 0$, line $\{(x_k, \lambda_k)\}$ convergent, to a *x(k)* and λ_k one that constitute Lagrange's solution and coefficient that corespondence to a subproblem.

Definition following to give convex requisite to form lagrange's modification.

Definition 2. To $\rho = 0$, form modifies Lagrange $\mathcal{L}(x, x_k, \lambda_k, 0)$ is convex.

Solving problem multi-stage stochastic nonlinear programs by use of lagrange's projection method depends on penalti's parameter ρ. If ρ too large therefore will be hard to find solution. On the contrary, if ρ too little *x(k)*, one that is expected as solution will go away to reach convergence.

Here after partision x as form x^L (x linear) and form x^N (x nonlinear). And partision also as $[B \;\; S \;\; N]$ with matrix *B* (basic) is matrix square and nonsingular, *S* (super basic) are matrix $m \times s$ by $0 \le s \le n - m$, and N (nonbasic) are residues column of matrix A., therefore constraints active becomes as follows;

$$\begin{bmatrix} B & S & N \\ & & I \end{bmatrix} \begin{bmatrix} x_B \\ x_S \\ x_N \end{bmatrix} = \begin{bmatrix} b \\ b_N \end{bmatrix}$$

where x_B, x_S, x_N called by basics's variable, superbasics and nonbasics what do accordingly with [*B, S, N*].

Note: basics's variable and superbasic is variable one free on bounds.

Theorem following to give that surety nonlinear's program have solution.

Theorem 3. Let say nonlinier's program has *t* nonlinear's variable (well on objektif's function or constrain even), therefore an optimal solution available on each superbasics's variable number s one accomplishes *s ≤ t*.

Proof . Let is variable non linear regular on appreciative optimal. Problem is rest is linear program for a basic's solution whatever available *(s=0)*. Its result is trivial if variable nonlinear is regarded as superbasic on early problem. If $s=t$, available variable nonlinear that current on bounds is nonbasic the so called. Therefore has $s \leq t$.

From Theorem 3 secure to mark sense optimal solutions so base 3 get to be made by defenition followings.

Definition 4. Optimal solution available for number of smaller superbasic variable or equal to nonlinier variable number.

Hereafter been given simple algorithm for multi-stage stochastic nonlinear programs.

Set 1. Let $K=0$, Choose some initial estimates x_0, y_0 and λ_0. Specify a penalty parameter $\rho \geq 0$ and convergence tolerance $\in_c > 0$.

Set 2. Given x_k, y_k, λ_k and ρ, solve the lineraly constrained subproblem (1.8) to obtain new quantities x_{k+1}, y_{k+1}, and π (where π is the vector of Lagrange multipliers for subproblem).

Set 3. Let λ_{K+1} = the first componenets of π.

Set 4. Test convergence (see Definition 5). If optimal, exit.

Set 5. Relinearize the constraints at x_{k+1}.

Set 6. Let $K= K+1$ and repeat from step 2.

Definition 5. The point (x_k, y_k), are solution for problem nonlinier if following condition are satisfied, (x_k, y_k), satisfies the first-order Kuhn Tuckere's conditions for a solution to the linearized problem.

From the theorems, definitions and algorithm that is given gets to be seen that lagrange projection method can utilized to solve multi-stage stochastic nonlinear programs.

Conclusions

A projected Lagrangian method is a very effective approach for solving medium-size nonlinear programming. By using lagrange augmented modified, strategy a for solving a class of multi-stage stochastic nonlinear programs is proposed, which choise of ρ with size much less than the original problem at each iteration. Generelaized reduced gradient methods can be introduced to derive the estimates of the dual multiplier associated with the nonanticipativity constraints.

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