

• Apply equivalence algorithms (for example, the Lie symmetry method) on DE, and obtain the general solutions as follows:

$$\tilde{u}(x, t; \varepsilon) := \rho(\exp(\varepsilon v_k)) u_0,$$

• Propose the suitable mother wavelet (as a test function) based on the symmetry groups and differential invariants. According to mother wavelet ψ , similar to the wavelet transforms definition, we have:

$$U(x, t) = \int \psi(x, t; \varepsilon) \cdot \rho(\exp(\varepsilon v_k)) u_0(x, t; \varepsilon) d\varepsilon,$$

This is a wavelet type transform that its kernel is an invariant solution of differential equation.

• For every generator vector field v_k from the symmetry groups, by considering the suitable values (0,1) for the parameters ε_i (this parameter for symmetry group G_i equals 1 and for every symmetry group G_j such that $j \neq i$ equals 0), and the correspondent version of mother wavelet ψ based on the differential invariants of the symmetry groups G_i (that's $\psi(x, t; \varepsilon_i)$), construct $U(x, t)$.

Indeed, $U(x, t)$ is a wavelet transform and on the other hand (for every invariant solution $\tilde{u}(x, t; \varepsilon)$), it is a wavelet invariant solution.

This method provides a link between harmonic and wavelet analysis and Lie symmetries. In fact, the new solutions can be constructed by integral kernel operators, where the kernels are invariant solutions (obtained by the symmetries of the equation) and test functions are suitable mother wavelets (constructed based on the differential invariants). Therefore, this method converges to the solution of differential equations.

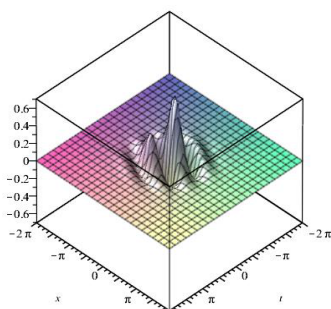
Examples

Here, we demonstrate WTTM by examples. We implement WTTM on the heat, KdV and GFKPP equations and obtain solutions. finally, WTTM results will be proposed. First, we should apply the Lie symmetry method on these equations, for more details and calculations [4]. The following table proposes the mother wavelets according to the differential invariants of symmetry groups: (Table 1).

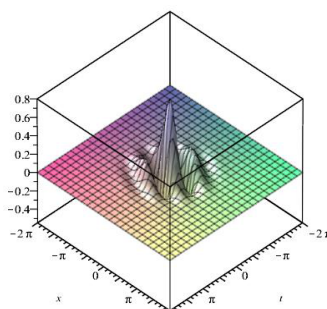
By the little computation, it can be seen that offered functions have

Symmetry group	Differential invariants	Mother wavelets
Translation	xct, u	$(4/5)\exp((x^2+t^2)/2)\sin(\pi(x-ct)/2)$
Scaling	$(x/t), (x/\sqrt{t}), (u/t^*)$	$(4/5)\exp(-(x^2+t^2)/2)\cos(\pi(xct)/2)$ $\exp(-(x^2+15t^2)/20)\cos(x/t)\sin(x/t)$
Galilean Boost	$t, u\exp(x^2/4t)$	$\exp(-(x^2+at^2)/b)\cos(x/t)\sin(x/t)$

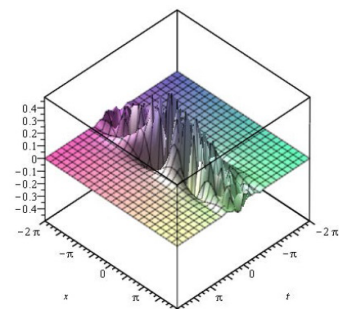
Table 1: Symmetry groups, Differential invariants and Mother wavelets.



(a) The graph of ψ_1



(b) The graph of ψ_2



(c) The graph of ψ_3

Figure 1: The graphs of mother wavelets.

properties (2)-(3) of the mother wavelets. Figure 1 shows the graphs of mother wavelets. The some properties are clear from this figure (some computations and plots done by the maple 2016).

We know that the general solution (from the Lie symmetry method) of the heat equation $u_t = u_{xx}$ as follows:

$$\tilde{u}(x, t; \varepsilon) = \frac{\exp\left(\varepsilon_3 - \frac{\varepsilon_5 x + \varepsilon_6 x^2 - \varepsilon_5^2 t}{1 + 4\varepsilon_6 t}\right) u\left(\frac{\exp(-\varepsilon_4)(x - 2\varepsilon_3 t)}{1 + 4\varepsilon_6 t}, \frac{\exp(-2\varepsilon_4)}{1 + 4\varepsilon_6 t} - \varepsilon_2\right)}{\sqrt{1 + 4\varepsilon_6 t}},$$

Thus for the translation symmetry group in the line of x , all ε_i (except $i=1$) are zero. So,

$$\tilde{u}(x, t; \varepsilon) = u(x - \varepsilon_1, t).$$

But for the vector field $v_1 = \partial_x + 2t\partial_t$, with the differential invariant $(x-2t)$ (the translation parameter $c=2$), we have:

$$\tilde{u}(x, t; \varepsilon) = u(x - 2\varepsilon_1, t - \varepsilon_2).$$

for the corresponding mother wavelet $\psi_1(x, t) = \frac{4}{5}\exp\left(-\frac{x^2+t^2}{2}\right)\sin\left(\frac{\pi(x-2t)}{2}\right)$, we get,

$$\psi_1(x, t; \varepsilon) = \frac{4}{5}\exp\left(-\frac{(x-2\varepsilon_1)^2 + (t-\varepsilon_2)^2}{2}\right)\sin\left(\frac{\pi(x-2\varepsilon_1-2t+2\varepsilon_2)}{2}\right)$$

Therefore:

$$\int_{\mathbb{R}^+} \int_{\mathbb{R}^+} \frac{4}{5}\exp\left(-\frac{(x-2\varepsilon_1)^2 + (t-\varepsilon_2)^2}{2}\right)\sin\left(\frac{\pi A}{2}\right)\tilde{u}(x, t; \varepsilon) d\varepsilon_1 d\varepsilon_2. \quad (5)$$

Where $A=(x-2t+2(\varepsilon_2-\varepsilon_1))$. Indeed U is the wavelet transform $W_{\psi_1}(x, t)$ on the related symmetry group v_1 that its invariant solution as below:

$$\tilde{u}(x, t; \varepsilon) = k\exp(-2A) + l,$$

By replacing this solution in the integral (5) and integration respect to $\varepsilon_1, \varepsilon_2$, we get to the wavelet invariant solution of heat equation based on $\psi_1(x, t)$. The calculation for mother wavelet ψ_2 is similar.

Now, let us consider mother wavelet ψ_4 related to the Galilean boost vector field $v_4=2t \partial_x x u \partial_u$ with differential invariant solution

