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The Very Cost Effective Graph Folding of the Join of Two Graphs

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Abstract

In this paper, we studied the very cost effective graph property for the join graph of two graphs. In general this is may or may not be a very cost effective graph. We obtained the conditions for the join graph of two graphs to be a very cost effective graph. First we proved that the join graph $P_n \vee P_m$ of path graphs is very cost effective graph if $n+m$ is an even number and is not if $n+m$ is an odd number. Then we proved that the join graph of any two cycle graphs C_n and C_m where n, m are both odd is very cost effective, and the join graph $P_n \vee C_n$ is a very cost effective graph if n is an odd number. Also we proved that the join graph $G_1 \vee G_2$ of two very cost effective graphs G_1 and G_2 is a very cost effective graph if $n(G_1)+n(G_2)$ is even. Finally we proved that the graph folding of the join graph of two very cost effective graphs not always very cost effective but this will be the case if the sum of the numbers of the vertices in the image of the graph folding is even.

Keywords: The join graph; Graph folding; Very cost effective graph

Introduction

A graph G=(V,E) is a nonempty, finite set of elements called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set of G is denoted by V (G) and the edge set of G is denoted by E(G). A simple graph is a graph with no loops and no multiple edges. If e={u, v} is an edge of a graph G, then u and v are adjacent vertices, while u and e are incident [1]. Two adjacent vertices are called neighbors of each other. The degree (valency) of a vertex v in a graph G is the number of edges incident to v [2]. A vertex of degree 0 in G is called an isolated vertex [3]. A vertex v is said to be even or odd, according to whether its degree in G is even or odd [4]. If all the vertices of a graph G have the same valency then it is called a regular graph [5]. The order of G, denoted n(G)=|V(G)|, is the number of vertices in G. The size of G, denoted m(G)=|E(G)|, is the number of edges in G [6]. A graph of order 1 is called a trivial graph, and a graph of order at least 2 is called a non-trivial graph. A graph of size 0 is called an empty graph. A nonempty graph has one or more edges [7]. A graph is said to be connected if every pair of vertices has a path connecting them [8], otherwise is called disconnected. For any vertex $v \in V(G)$, the open neighborhood of v is the set $N(v)=\{ u \in V(G) \mid uv \in E(G)\}$, and the closed neighborhood of v is the set $N[v]=\{N(v) \cup \{ v \}\}$. For a set $S \subseteq V(G)$, its open neighborhood $N(S)=\cup_{v \epsilon S} N(v)$, and its closed neighborhood is $N[S]=N(S) \cup S$. A path Pn is graph in which any two vertices are connected by exactly one edge with two vertices of degree 1, and the other n-2 vertices of degree 2.

Definition (1.1)

Let G_1 and G_2 be simple graphs and f: $G_1 \to G_2$ be a continuous function. Then f is called a graph map, if

1. For each vertex $v \epsilon V(G_1)$, f(v) is a vertex in $V(G_2)$.
2. For each edge $e \epsilon E(G_1)$, $\dim(f(e)) \le \dim(e)$, [4].

Definition (1.2)

A graph map f: $G_1 \to G_2$ is called a graph folding if f maps vertices to vertices and edges to edges, i.e., for each vertex $v \epsilon V(G_1)$, f(v) is a vertex in $V(G_2)$ and for each edge $e \epsilon E(G_1)$, f(e)is an edge in $E(G_2)$ [2].

Cost effective and very cost effective sets in graphs were introduced [6] and studied further [7]. Very cost effective bipartitions were also first introduced [6] and were motivated by the studies of unfriendly partitions [1].

Definition (1.3)

A vertex v in a set S is said to be cost effective if it is adjacent to at least as many vertices in V\S as in S, that is, $|N(v) \cap S| \le |N(v) \cap (V\setminus S)|$. A vertex v is very cost effective if it is adjacent to more vertices in V\S than in S, that is, $|N(v) \cap S| < |N(v) \cap (V\setminus S)|$. A set S is (very) cost effective if every vertex $v \in S$ is (very) cost effective [6].

Definition (1.4)

A bipartition π={S, V\S} is called cost effective if each of S and V\S is cost effective, and π is very cost effective if each of S and V\S is very cost effective. Graphs that have a (very) cost effective bipartition are called (very) cost effective graphs [7].

Note that not every graph has a very cost effective bipartition, e.g., the cycle and the complete graphs of odd orders are not very cost effective.

Definition (1.5)

If G_1 and G_2 are vertex-disjoint graphs. Then the join, $G_1 \vee G_2$, of G_1 and G_2 is a super graph of G_1+G_2, in which each vertex of G_1 is adjacent to every vertex of G_2. The vertex set $V(G_1 \vee G_2)=V_1 \cup V_2$, [5].

The Join Graph of Very Cost Effective Graph

We will study the very cost effective graph property for the join graph of two graphs. First we pay attention to path graphs where any path graph is a very cost effective graph [6]. It should be noted that

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the join graph $P_n \vee P_m$ is not necessary a very cost effective graph. For example $P_2 \vee P_3$ and $P_4 \vee P_5$ are not very cost effective graphs (Figure 1).

In general this is true if n+m is an odd number, as it is proved in theorem (2.3). The following two lemmas give very cost bipartitions for path graphs P_n, where n is odd or even.

Lemma (2.1)

Any path P_i, i=1,2, …, n where n is even has the only very cost bipartition $\pi=\{S, V\backslash S\}$, where $S=\{v_1, v_3, \dots, v_{n-1}\}$ and $V\backslash S=\{v_2, v_4, \dots, v_n\}$. In this case $n(S)=n(V\backslash S)$.

Proof:

Let u be a vertex in S, then $|N(u) \cap S)|=\emptyset$, $|N(u) \cap (V\backslash S)|=1$ or 2 (1 for an end vertex and 2 otherwise) and hence u is very cost vertex in S. Also, we can prove that $V\backslash S$ is a very cost effective set. Hence π is a very cost bipartition (Figure 2a).

Lemma (2.2)

Any path P_i, i=1,2, …, n where n is odd has the only very cost bipartitions $\pi_1=\{S_1, V_1\backslash S_1\}$, where $S_1=\{v_1, v_3, \dots, vn\}$ and $V_1\backslash S_1=\{v_2, v_4, \dots, vn_{-1}\}$ or $\pi_2=\{S_2, V_2\backslash S_2\}$, where $S_2=\{v_2, v_4, \dots, vn_{-1}\}$, $V_2\backslash S_2=\{v_1, v_3, \dots, vn\}$. In this case either $n(S_1)=n(V_1\backslash S_1)+1$ or $n(V_2\backslash S_2)=n(S_2)+1$.

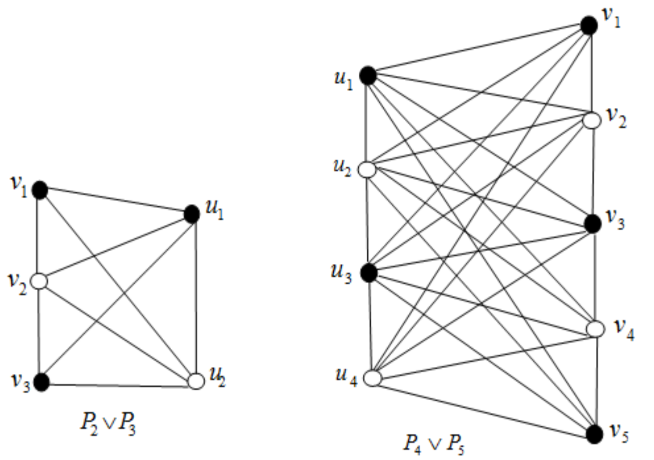

Figure 1: $P_2 \vee P_3$ and $P_4 \vee P_5$ cost effective graphs.

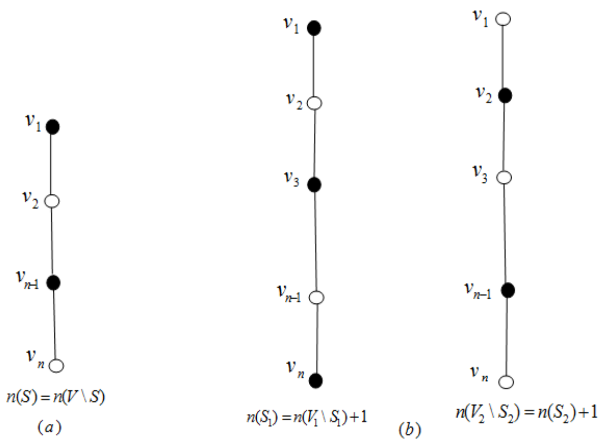

Figure 2: Bipartition cost effective graphs.

Proof:

By the same procedure as lemma (2.1), we can prove this lemma, Figure 2b.

Theorem (2.3)

The join of any path graphs P_n and P_m where $n+m$ is an odd number is not a very cost effective graph.

Proof:

Consider two path graphs P_n and P_m with very cost effective bipartitions $\pi_1=\{S_1, V_1\backslash S_1\}$ and $\pi_2=\{S_2, V_2\backslash S_2\}$, where $V_1=V(P_n)$ and $V_2=V(P_m)$. Without loss of generally, we may assume n is even and m is odd, then $n(S_1)=n(V_1\backslash S_1)$ and let $n(S_2)=n(V_2\backslash S_2)+1$.

Let u be an end vertex in S_1, then u is adjacent to more vertices of $V_1\backslash S_1$ than in S_1 by one. Now consider the join graph $Pn \vee P_m$ with the bipartition $\pi=\{S, V\backslash S\}$ where $S=S_1 \cup S_2$ and $V\backslash S=(V_1\backslash S_1) \cup (V_2\backslash S_2)$, the vertex u in this case will be adjacent to all the vertices of P_m such that $n(N(u) \cap S)=n(S_2)$ and $n(N(u) \cap V\backslash S)=n(V_2\backslash S_2)$. But $n(S_2)=n(V_2\backslash S_2)+1$. This mean that the vertex u is adjacent to the same number of the vertices of S_2 and $V_2\backslash S_2$, and thus the join graph $P_n \vee P_m$ is not a very cost effective graph.

Theorem (2.4)

The join of any path graphs P_n and P_m where $n+m$ an even number is a very cost effective graph.

Proof:

Consider two path graphs P_n and P_m with very cost effective bipartitions $\pi_1=\{S_1, V_1\backslash S_1\}$ and $\pi_2=\{S_2, V_2\backslash S_2\}$, respectively. We have two cases.

Case 1: n and m are both even

In this case $n(S_1)=n(V_1\backslash S_1)$ and $n(S_2)=n(V_{2\backslash}S_2)$. For any vertex $u \in Pn$, u is adjacent to more vertices in $V_1\backslash S_1$ than in S_1. But u in the join graph $Pn \vee P_m$ will be adjacent to the same numbers of vertices of S_2 and $V_2\backslash S_2$, then u is a very cost effective vertex. This also the case for any vertex $v \in P_m$. Then the join graph $Pn \vee P_m$ is a very cost effective graph.

Case 2: n and m are both odd.

Consider the very cost effective bipartitions π_1 and π_2 where $n(S_1)=n(V_1\backslash S_1)+1$ and $(V_2\backslash S_2)=n(S_2)+1$. Let $u \in S_1$ so u is adjacent to more vertices in $V_1\backslash S_1$ than in S_1 by at least one (1 or 2).

But in the join graph $P_n \vee P_m$, where $S=S_1 \cup S_2$ and $V\backslash S=(V_1\backslash S_1) \cup (V_2\backslash S_2)$, the vertex u will be adjacent to all the vertices of $V_2\backslash S_2$ and S_2, so u is adjacent to more vertices of $V\backslash S$ than in S. Thus u is very cost effective vertex. Now, let $v \in (V_1\backslash S_1)$, so v is adjacent to more vertices in S_1 than in $V_1\backslash S_1$ by two. But in the join graph $P_n \vee P_m$ the vertex v will be adjacent to more vertices of S than in $V\backslash S$, so v is a very cost effective vertex, and consequently the join graph $P_n \vee P_m$ is a very cost effective graph.

Theorem (2.5)

The join graphs $P_2 \vee P_4$ and $P_3 \vee P_5$ are very cost effective graphs (Figure 3).

In the following we would like to answer the following questions

1. Is every join graph $G_1 \vee G_2$ is a much cost?
2. If G_1 is very cost effective, is $G_1 \vee G_2$ very cost effective for all graphs?

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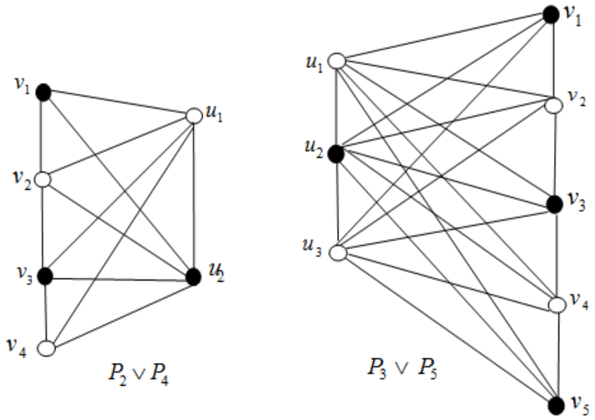


Figure 3: $P_2 \vee P_4$ and $P_3 \vee P_5$ are very cost effective graphs.

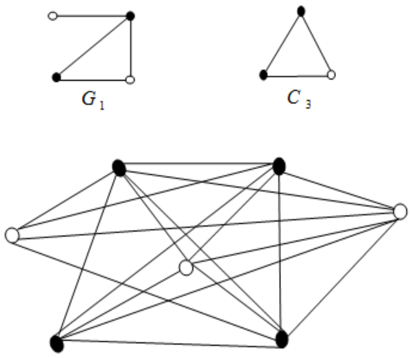


Figure 4: $G_1 \vee C_3$ is not very cost effective.

3. If G_1 and G_2 are both very cost effective, is $G_1 \vee G_2$ very cost effective?

The answer of the first question is, in general, no, i.e., the join $G_1 \vee G_2$ of non-trivial connected graphs G_1 and G_2 may or may not a very cost effective graph. For example, the join graph $G_1 \vee C_3$ shown in Figure 4 is not a very cost effective graph while the join graph $C_3 \vee C_3$ shown in Figure 5 is a very cost effective graph.

The answer of the first question is yes for the following special case of graphs. Before discussing this case it should be noted that no cycle graph C_{2n+1} of odd order is very cost effective [6].

Lemma (2.6)

For a cycle graph Cn, where n is odd the bipartition $\pi=\{S, V\backslash S\}$, where $S=\{v_1, v_2, v_4, \ldots, vn_{-1}\}$ and $V\backslash S=\{v_3, v_5, \ldots, vn\}$ is cost effective. In this case $n(S)=n(V\backslash S)+1$.

It is easy to prove the vertex $v_1 \in S$ is cost effective while the other vertices in both S and $V\backslash S$ are very cost effective vertices and hence π is a cost effective bipartition (Figure 6).

Note that the bipartition $\pi=\{V\backslash S, S\}$ is also a cost bipartition.

Theorem (2.7)

The join graph of any two cycle graphs Cn and C_m where n, m are both odd is a very cost effective graph.

Proof:

Let Cn and C_m be any two cycle graphs where n, m are both odd.

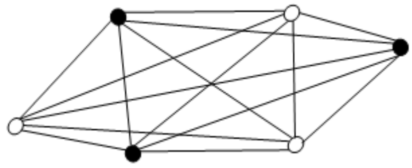


Figure 5: $C_3 \vee C_3$ is a very cost effective.

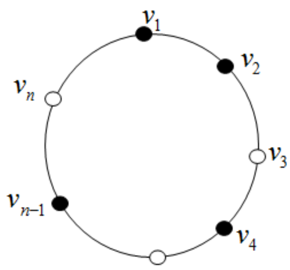


Figure 6: $v_1 \in S$ is cost effective.

Choose the cost effective bipartitions $\pi_1=\{S_1, V_1\backslash S_1\}$ and $\pi_2=\{S_2, V_2\backslash S_2\}$ such that $n(S_1)=n(V_1\backslash S_1)+1$ and $n(V_2\backslash S_2)=n(S_2)+1$. Now consider the join graph $C_n \vee C_m$ with the cost bipartition $\pi=\{S, V\backslash S\}$, where $S=S_1 \cup S_2$ and $V\backslash S=(V_1\backslash S_1) \cup (V_2\backslash S_2)$. Now, let $u \in S_1$, if u is a very cost effective vertex, then u must be adjacent to more vertices of $V_1\backslash S_1$ than in S_1 and since u is adjacent to all the vertices of S_2 and $V_2\backslash S_2$ and $n(S_2)=n(V_2\backslash S_2)-1$, then u is still a very cost effective vertex in the join graph $C_n \vee C_m$. Now, suppose $u \in S_1$ is a cost effective vertex, then u will be adjacent to at least as many vertices in $V_1\backslash S_1$ as in S_1 and since u is adjacent to all the vertices of S_2 and $V_2\backslash S_2$ and $n(V_2\backslash S_2)=n(S_2)+1$. Thus u is adjacent to more vertices in S than in $V\backslash S$ by at least one and consequently u is a very cost effective vertex. Now, let $u \in (V_1\backslash S_1)$, then u must be adjacent to more vertices in S_1 than in $V_1\backslash S_1$ certainly by at least two and since u is adjacent to all the vertices of S_2 and $(V_2\backslash S_2)$ and $n(S_2)=n(V_2\backslash S_2)-1$, then u is still a very cost effective vertex in the join graph $C_n \vee C_m$. Also, we can prove that any vertex of S_2 or $V_2\backslash S_2$ is a very cost effective vertex and hence π is a very cost bipartition, i.e., $C_n \vee C_m$ is a very cost effective graph.

Theorem (2.8)

The join graph $C_3 \vee C_5$ is a very cost effective graph (Figure 7).

Now let G_1 be a very cost effective graph and G_2 any non-trivial connected graph. The join graph of $G_1 \vee G_2$ is not always very cost effective; the following example illustrates this fact.

Theorem (2.9)

The join graph $C_3 \vee C_4$ shown in Figure 8a is not a very cost effective graph while the join graph $P_3 \vee C_3$ is very cost effective (Figure 8b).

Thus the answer of the second question is, in general, no. The answer is yes in the following case.

Theorem (2.10)

The join graph of any path P_n and cycle C_n where n is odd is a very cost effective graph.

Proof:

By the same procedure as in theorem (2.7) we can prove this theorem.

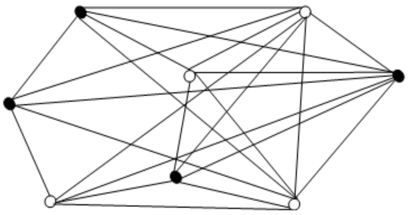
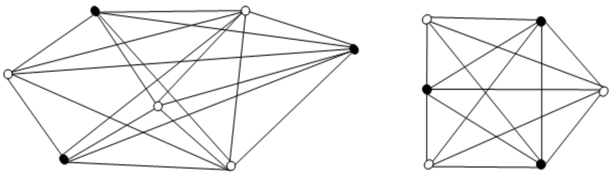


Figure 7 : $C_3 \vee C_3$ is very cost effective.



(a) (b)

Figure 8: (a) $C_3 \vee C_4$ is not cost effective; (b): $P_3 \vee C_3$ is very cost effective.

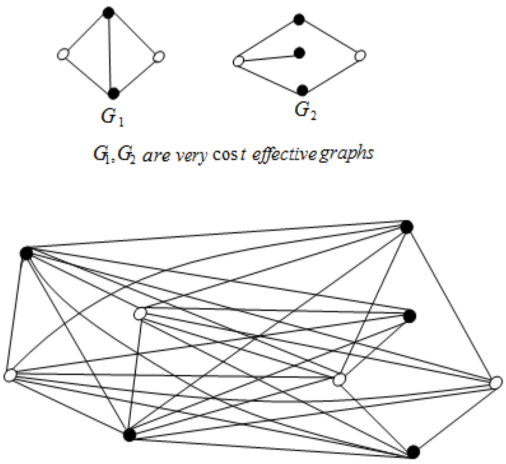


Figure 9: $G_1 \vee G_2$ is not very cost effective.

Now, we come to the third question. The answer of the third question is also, in general, no, i.e., if G_1 and G_2 are two very cost effective graphs, then the join graph $G_1 \vee G_2$ may or may not be very cost effective. For example, if G_1 and G_2 are the very cost effective graphs shown in Figure 9, then the join graph $G_1 \vee G_2$ is not very cost effective.

While the join graph $K_4 \vee K_{,22}$ is a very cost effective graph (Figure 10).

The following theorem gives the condition for which the join graph of two very cost effective graphs is very cost effective.

Theorem (2.11)

The join graph of two non- trivial connected very cost effective graphs G_1 and G_2 is a very cost effective graph if $n(G_1)+n(G_2)$ is even.

Proof:

Let G_1 and G_2 be any non-trivial connected graphs, we have two cases

(1) $n(G_1)$ and $n(G_2)$ are both even. Then each graph has a very cost effective bipartition. Let $\pi_1=\{S_1, V_1\backslash S_1\}$ and $\pi_2=\{S_2, V_2\backslash S_2\}$, be such that

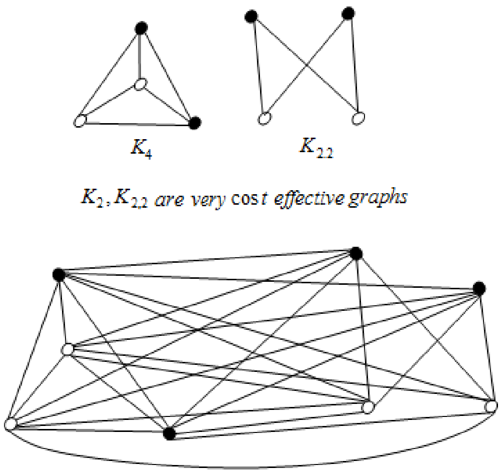


Figure 10: $K_4 \vee K_{,22}$ is a very cost effective graph.

$n(S_1)=n(V_1\backslash S_1)$ and $n(S_2)=n(V_2\backslash S_2)$. Now consider the join graph $G_1 \vee G_2$ with the cost bipartition $\pi=\{S, V\backslash S\}$, where $S=S_1 \cup S_2$ and $V\backslash S=(V_1\backslash S_1) \cup (V_2\backslash S_2)$. Let $u \in S_1$, then u is adjacent to more vertices of $V_1\backslash S_1$ than in S_1 and since u is adjacent to all the vertices of S_2 and $V_2\backslash S_2$, then u is still a very cost effective vertex in the join graph $G_1 \vee G_2$. Also we can prove that any vertex of $V_1\backslash S_1$ or S_2 or $V_2\backslash S_2$ is a very cost effective vertex and hence π is a very cost effective bipartition, i.e., $G_1 \vee G_2$ is a very cost effective graph.

(2) $n(G_1)$ and $n(G_2)$ are both odd. Suppose that $\pi_1=\{S_1, V_1\backslash S_1\}$ and $\pi_2=\{S_2, V_2\backslash S_2\}$ are very cost effective bipartitions such that $n(S_1)=n(V_1\backslash S_1)+1$ and $n(V_2\backslash S_2)=n(S_2)+1$. Let $\pi=\{S, V\backslash S\}$ be a cost bipartition of the join graph $G_1 \vee G_2$, where $S=S_1 \cup S_2$ and $V\backslash S=(V_1\backslash S_1) \cup (V_2\backslash S_2)$. Now, let $u \in S_1$, then u must be adjacent to more vertices of $V_1\backslash S_1$ than in S_1 and since u is adjacent to all the vertices of S_2 and $V_2\backslash S_2$ and $n(S_2)=n(V_2\backslash S_2)-1$, then u is still a very cost effective vertex in the join graph $G_1 \vee G_2$. Now, let $u \in V_1\backslash S_1$, then u must be adjacent to more vertices in S_1 than in $V_1\backslash S_1$ certainly by at least two and since u is adjacent to all the vertices of S_2 and $V_2\backslash S_2$, then u is adjacent to more vertices in S than in $V\backslash S$ by at least one and u is a very cost effective vertex in the join graph $G_1 \vee G_2$. Also we can prove that any vertex of S_2 or $V_2\backslash S_2$ is very cost effective vertex and hence π is a very cost effective bipartition, i.e., $G_1 \vee G_2$ is a very cost effective graph.

Theorem (2.12)

1) The join graph $C_4 \vee W_{1,3}$ of the very cost effective graphs C_4 and $W_{1,3}$ is once again is a very cost effective graph (Figure 11).

2) Let G_1 and G_2 be the very cost effective graphs shown in Figure 12. Then the join graph $G_1 \vee G_2$ is also very cost effective graph (Figure 12).

The Join Graph Folding of Very Cost Effective Graphs

In this section we study very cost effective graph folding of a new graph obtained by the join of two other graphs.

Definition (3.1)

Let $G_1=(V_1, E_1)$, $G_2=(V_2, E_2)$, $G_3=(V_3, E_3)$ and $G_4=(V_4, E_4)$ be graphs. Let $f: G_1 \to G_3$ and $g: G_2 \to G_4$ be graph maps. A join map $f^V g: G_1 V G_2 \to G_3 V G_4$ is a map defined by

$$\text{For each vertex } v \in V_1 \cup V_2, (fVg)(v) = \begin{cases} f(v), \text{if } v \in V_2 \\ g(v), \text{if } v \in V_2 \end{cases}$$

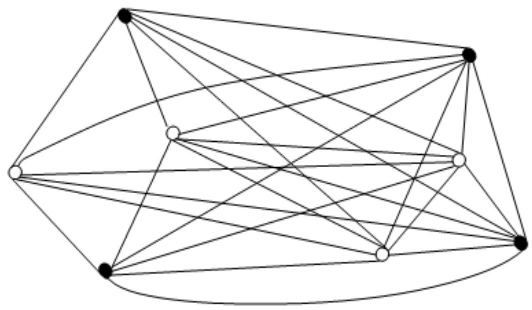


Figure 11: $C_4 \vee W_{1,3}$ of the very cost effective.

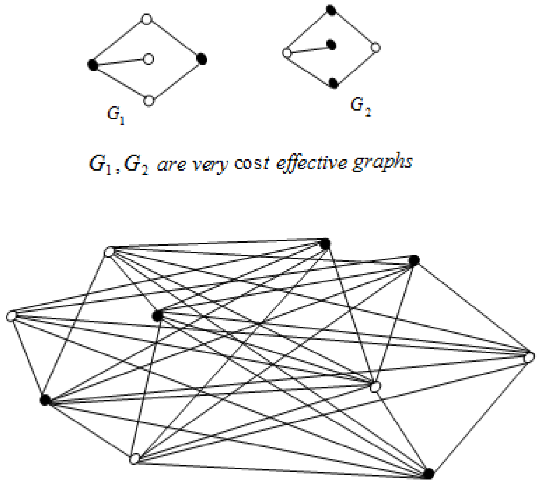


Figure 12: $G_1 \vee G_2$ is also very cost effective graph.

For each edge $e=(v_1,v_2)$, $v_1 \in V_1$ and $v_2 \in V_2$, $(f\vee g)\{e\}=\{f(v_1),g(v_2)\} \in G_3 \vee G_4$.

If $e=(u_1, v_1)\in E_1$ then $(f \vee g)\{e\}=\{f(u_1), f(v_1)\}$, also if $e=(u_2, v_2)\in E_2$, then $(f \vee g)\{e\}=(f \vee g)\{(u_2, v_2)\}=\{ g(u_2), g(v_2)\}$ [3].

Definition (3.2)

A graph folding $f: G_1 \to G_2$ between two graphs G_1 and G_2 is a very cost effective graph folding if the image $f(G_1)=H \subseteq G_2$ is a very cost effective graph [8].

It should be noted that the image of a graph folding $f: G\to G$ of a very cost effective graph G may or may not be a very cost effective graph

e.g., if G is the cycle graph C_4 where $V(C_4)=\{u_1, u_2, u_3, u_4\}$ and $E(C_4)=\{e_1, e_2, e_3, e_4\}$ then the graph folding $f: C_4\to C_4$ defined by $f\{u_1, u_2, u_3, u_4\}=\{u_3, u_2, u_3, u_4\}$ and $f\{e_1, e_2, e_3, e_4\}=\{e_4, e_3, e_3, e_4\}$ is a very cost effective graph folding since the image is P_3 (Figure 13).

From now on the omitted vertices or edges will be mapped into themselves. Also if G is the very cost effective graph shown in Figure 14 where $V(G)=\{v_1, v_2, v_3, v_4\}$ and $E(G)=\{e_1, e_2, e_3, e_4, e_5\}$ then the graph folding $g: G\to G$ defined by $g\{v_1\}=\{v_3\}$ and $g\{e_1, e_2\}=\{e_4, e_3\}$ is not a very cost effective.

Once again if f and g are both very cost effective graph foldings, the join map $(f \vee g)$ may or may not be a very cost effective graph folding. The following two theorems illustrate this.

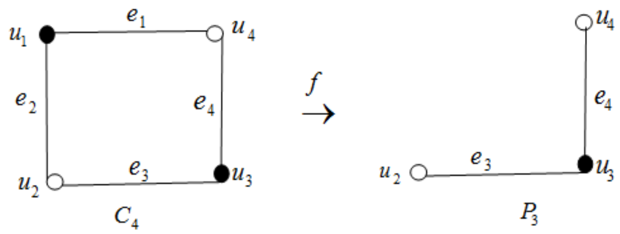


Figure 13: P_3 is also very cost effective graph.

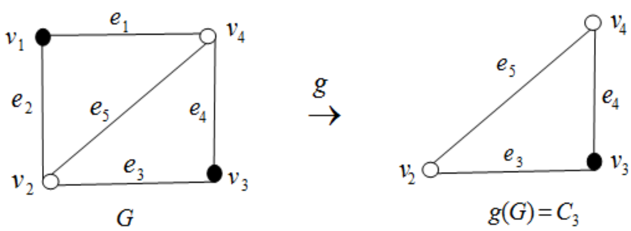


Figure 14: G is not cost effective graph.

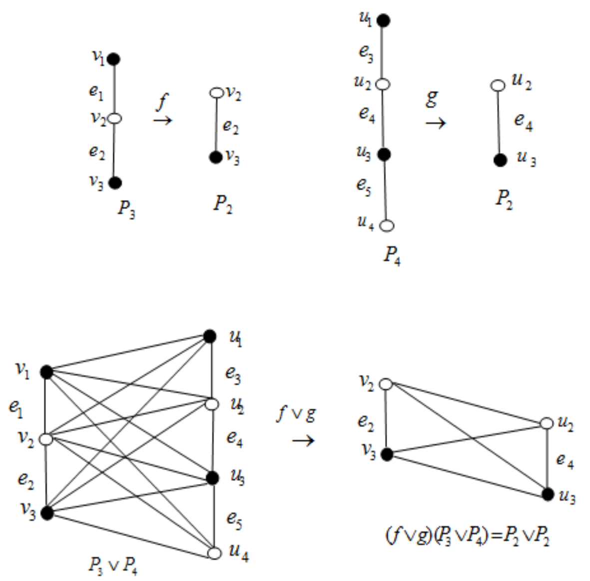


Figure 15: $f \vee g$: $G_1 \vee G_2 \to G_1 \vee G_2$ is a very cost effective graph.

Theorem (3.3)

Let $G_1=P_3$ and $G_2=P_4$ as shown in Figure 15. Let $f : G_1\to G_1$ and $g: G_2\to G_2$ be the very cost effective graph foldings defined by $f\{v_1\}=\{v_3\}$, $f\{e_1\}=\{e_2\}$ and $g\{u_1, u_4\}=\{u_3, u_2\}$ and $g\{e_3, e_5\}=\{e_4, e_4\}$. The join graph folding $f \vee g$: $G_1 \vee G_2 \to G_1 \vee G_2$ defined by $(f\vee g)\{u_1, u_4, v_1\}=\{u_3, u_2, v_3\}$ is a very cost effective graph folding.

Theorem (3.4)

Let G_1 and G_2 be any connected graphs and f, g are very cost effective graph foldings of G_1 and G_2, respectively. Then $(f\vee g)(G_1 \vee G_2)$ is a very cost effective graph folding if no. $V(f(G_1))$+no. $V(g(G_2))$ is even. The proof is almost the same as the proof of theorem (2.11).

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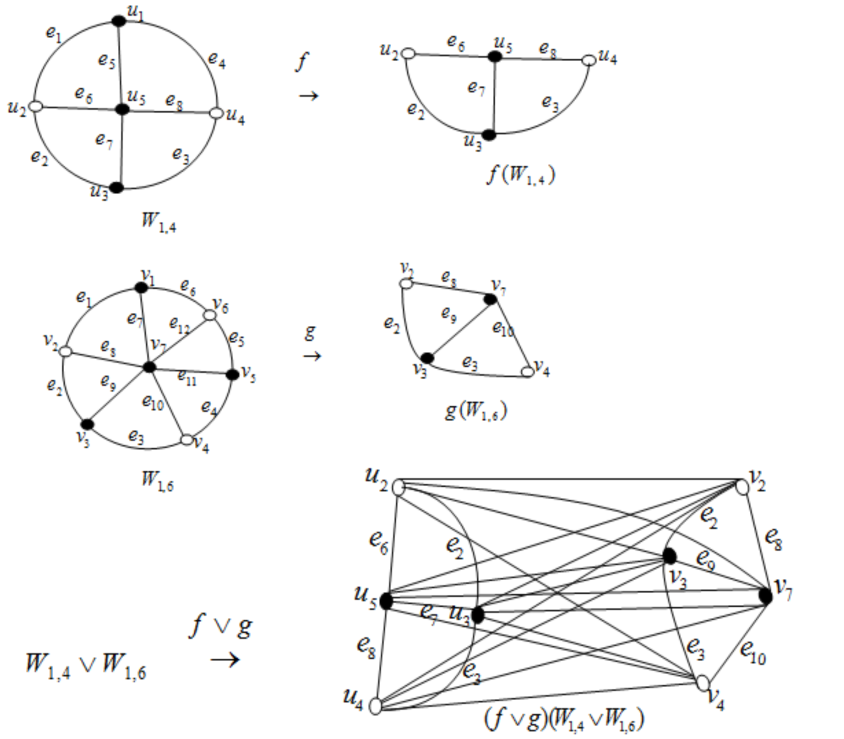

Figure 16: f ∨g: $W_{1,4} \vee W_{1,6} \rightarrow W_{1,4} \vee W_{1,6}$ is a very cost effective graph.

Theorem (3.5)

Consider the wheel graphs $W_{1,4}$ and $W_{1,6}$ shown in Figure 16. Let f: $W_{1,4} \rightarrow W_{1,4}$ and g: $W_{1,6} \rightarrow W_{1,6}$ be the very cost effective graph folding defined By f{u_1}={u_3} and f{e_1, e_4, e_5}={e_2, e_3, e_7} and g{v_1, v_5, v_6}={v_3, v_3, v_4} and g{e_1, e_4, e_5, e_6, e_7, e_{11}, e_{12}}={e_2, e_3, e_4, e_3, e_9, e_9, e_{10}}. Then The join graph folding f ∨g: $W_{1,4} \vee W_{1,6} \rightarrow W_{1,4} \vee W_{1,6}$ is a very cost effective graph folding (Figure 16).

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