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Two News Approximations to Standard Normal Distribution Function

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Abstract

This paper, presents two news approximations to the Cumulative Distribution Function (CDF). The first approximation 22 improves the accuracy of approximation given by Hart. In this first new approximation, we reduce the maximum absolute error (MAE) from, 0.004304 to 2.707*e*-004.For the second new approximation, Aludaat and Alodat was reduce the (MAE) from, 0.00314 to 0.001972. In this paper, we reduce the (MAE) to 0.001623. However, the first approximation is more accurate and its inverse is hard to calculate. The second new approximation is less accurate, but his inverse is easy to calculate.

Keywords: Cumulative distribution function; Maximum absolute error

Introduction

The cumulative distribution function (CDF) play an important role in financial mathematics and especially in pricing options with Black-Scholes Model. The probability density function defined by $\varphi(x)=\frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$. The (C.D.F.) function denoted $N(z)$ given by $N(z)=P(Z\le z)=\int_{-\infty}^{z}\varphi(x)dx$.

This function does not have a closed form. His evaluation is an expensive task. For evaluate the (CDF) at a point z we need compute the integral under the probability density function (PDF). In much research we find approximations, with a closed form, for the area under the standard normal curve. Otherwise, we need consulting Tables of cumulative standard normal probabilities. Hence, in the literature, we find several approximations to this function from Polya [1] to Yerukala [2].

Improving Hart Approximation

We consider the case of $z \ge 0$. For $z < 0$, $N(z)=1-N(-z)$.

The original Hart's approximation given by [3]:

$$N_{Hart}(z)\approx 1-\frac{1}{\sqrt{2\pi}}\frac{e^{-\frac{z^2}{2}}}{z+0.8e^{-0.4z}}$$

The absolute error denoted by $E(z)=|N(z)-N_{Hart}(z)$.

Figure 1; show the graph of E as function of $0 \le z \le 5$ (for ≥ 5, $max_z\, E(z) \le 4.4e-009$)

Hence, for $0 \le z \le 10$, we have $max_z\, E(z)=0.004303453286189 \le 0.004304$.

This formula have the form: $N_{Hart}(z)\approx 1-a\frac{e^{-bz^2}}{z+ce^{-dz}}$

We find a, b, c, and d that the MEA was the smallest possible. Numerical experiences show that the corresponding values of thus two parameters are (Tables 1 and 2):

a=0.39894, b=0.5078, c=0.79758 and d=0.4446

Hence, for $0 \le z \le 5$, we have the new approximation to the (CDF) given by:

$$\hat{N}_{Malki}(z)=1-0.39894\frac{e^{-0.5078z^2}}{z+0.79758e^{-0.4446z}}.$$

$$\max_z\left|\hat{N}_{Malki}(z)-N(z)\right|=0.000271664104197\le 0.000272.$$

Figure 2 gives the curves of original absolute error and the new absolute error

Improving Polya Approximation

The second approximation have the following form (Figures 3 and 4)

$N_{Polya}(z)=\frac{1}{2}\left\{1+\sqrt{1-e^{-az^2}}\right\}$ where $a=\frac{2}{\pi}$ with maximum absolute error.

$$\max_z\left|N_{Polya}(z)-N(z)\right|=0.003138181653387.$$

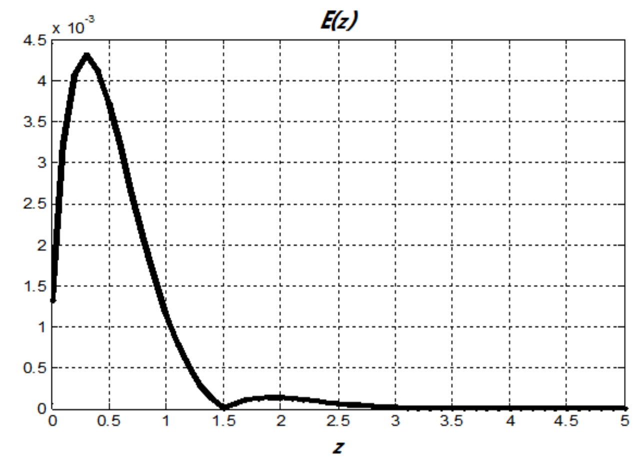

Figure 1: Hart's Error as function of variable z for 0≤z≤5, shows that M.E.A. occurs in [0.2;0.5].

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z	N(z)	$N_{Hart}(z)$	E(z)
0.2	0.5793	0.5833	0.0041
0.3	0.6179	0.6222	0.0043
0.4	0.6554	0.6595	0.0041
0.5	0.6915	0.6952	0.0037

Table 1: Shows that M.E.A. occurs in [0.2;0.5].

Parameter a	M.A.E.
2/π(Polya)	0.003138181653387
$\sqrt{\pi/8}$ (Aludaat)	0.001971820656170
0.623060	0.001623086812595
0.62306179(Malki)	0.001622801925711
0.623062	0.001622820064148

Table 2: This chose a=0.62306179 is justified by the following table.

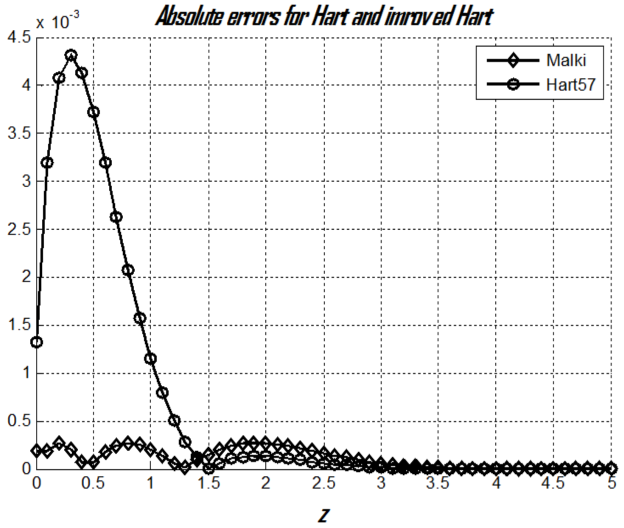

Figure 2: Comparison the original absolute error with a new absolute error.

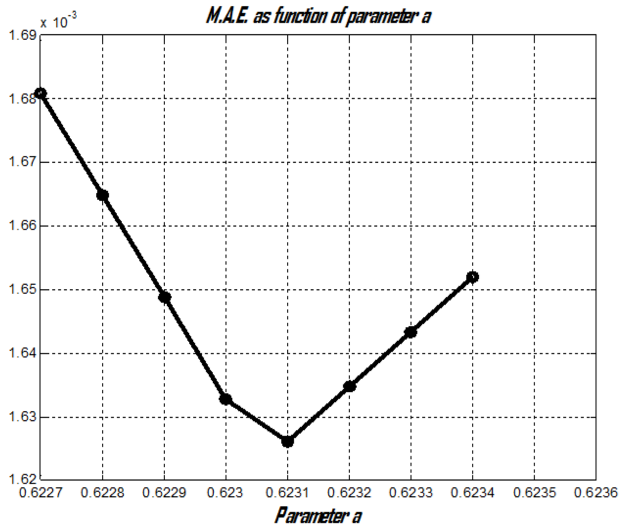

Figure 3: The optimal parameter a∈[0.6230;0.6232].

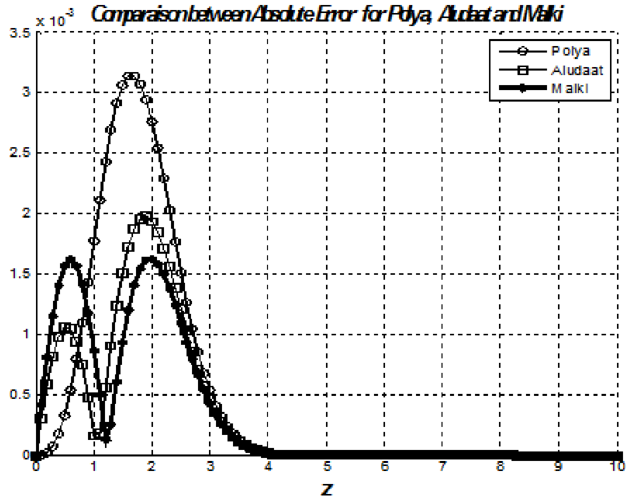

Figure 4: Curve of M.E.A as function of variable z.

Aludaat and Alodat [4] proposed the same formula with $a=\sqrt{\frac{\pi}{8}}$ instead of $a=\frac{2}{\pi}$

They obtain the maximum absolute error $max_z|N_{Aludaat}(z) - N(z)$=0.001971820656170. In this paper, we chose a=0.62306179. In this case the MAE is $max_z|N_{Malki}(z) - N(z)$=0.001622801925711 [5-9].

Conclusion

We have proposed two approximations to the cumulative distribution function of the standard normal distribution. The first approximation improve the Hart's formula in accuracy. The second new approximation improve the improving Polya's formula given by Aludaat KM and Alodat MT [4].

We hope an application of the first approximation to option pricing of a Call European option based on Black-Scholes formula.

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