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Undetermined Functions Method for Solving First Order Differential Equations

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Abstract

Most authors of differential equations used integrating factor to solve linear first or-der ordinary differential equations. In this paper, we introduce undetermined functions method to solve linear first order ordinary differential equations. Moreover, we derive solution method for solving linear first order ordinary differential equations without applying exactness condition.

Keywords: Equivalent; Differential equations; Derivatives; Integrating factor

Introduction

In this paper, we introduce undetermined functions method to solve linear first order ordinary differential equations.

A differential equation is an equation that relates an unknown function and one or more of its derivatives of with respect to one or more independent variables [1]. If the unknown function depends only on a single independent variable, such a differential equation is ordinary differential equation. If the unknown function depends only on many independent variables, such a differential equation is partial differential equation. An ordinary differential is linear if it is linear in the unknown function and its derivatives that involve in it. The order of an ordinary differential equation is the or-der of the highest derivative that appears in the equation [2]. Moreover, differential equations are classified into two main categories. The first one is ordinary differential equations and the other is partial differential equations.

A solution of a differential equation in the unknown function y and the independent variable x on the interval I is a function y(x) that satisfies the differential equation identical for all x in I [2]. A solution of a differential equation with arbitrary parameters is called a general solution. A solution of a differential equation that is free of arbitrary parameters is called a particular solution [2]. A solution in which the de-pendent variable is expressed solely in terms of the independent variable and constants is said to be an explicit solution. A relation *G(x,y)* is said to be an implicit solution of an ordinary differential equation on an interval I, provided there exists at least one function f that satisfies the relation as well as the DE on I [1]. Moreover, solution of differential equations is classified as trivial and non-trivial solutions, general and particular solutions and explicit and implicit solutions.

Our objective is to introduce linear first order ordinary differential equations and their solution method. Therefore, first we define linear first order ordinary differential equations. Finally, we derive solution method to solve linear first order ordinary differential equations.

Motivation

Research questions

1) Does solution method for solving first order linear ordinary differential equations in general exist?

2) Can we solve first order linear ordinary differential equations without applying integrating factor?

There exists method for solving linear first order ordinary differential equations by applying integrating factor [3]. One can solve linear first order ordinary differential equations without applying integrating factor. In this manuscript, we derive solution method for solving linear first order ordinary differential equations without applying integrating factor.

Linear First Order Ordinary Differential Equation and its Solution

The linear first order ordinary differential equation with unknown dependent variable y and independent variable x is defined by

$$a_0(x)y + a_1(x)y^{(1)} = g(x). \quad (4.1)$$

The general solution of the equation in equation 4.1 is given by

$$y = \frac{\int \frac{\mu(x)g(x)}{a_1} dx}{\mu(x)} \quad (4.2)$$

where $\mu(x) = \exp(\int (\frac{a_0(x)}{a_1(x)})dx)$ [3].

Here $\mu(x) = \exp(\int (\frac{a_0(x)}{a_1(x)})dx)$ is called integrating factor of equation in 4.1.

Derivation of Undetermined Functions Method for Solving Linear First Order Ordinary Differential Equations

Let's consider the equation

$$p(x)y = s(x) \quad (5.1)$$

We differentiate both sides of equation in 5.1 to get that

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$$p^{(1)}(x)y + p(x)y^{(1)} = s^{(1)}(x) \quad (5.2)$$

we observe that

$$y = \frac{s(x)}{p(x)} \quad (5.3)$$

is solution of equation in 5.1. Therefore, y in equation 5.3 is solution of 5.4. Let equation in 4.1 be linear first order ordinary differential equation.

Let *h(x)* be function of *x*.

Let's assume this function as undetermined function that satisfies

$$\frac{h^{(1)}(x)}{h(x)} = \frac{a_0(x)}{a_1(x)} \quad (5.4)$$

Also, let's assume that f (x) as undetermined function that satisfies

$$\frac{f^{(1)}(x)}{h(x)} = \frac{g(x)}{a_0(x)} \quad (5.5)$$

where *h(x)* is a function that satisfies equation in 5.4

Thus, equation in 4.1 is equivalent to

$$h^{(1)}(x)y + h(x)y^{(1)} = f^{(1)}(x) \quad (5.6)$$

where *h(x)* is a function that satisfies equation in 5.4

The equation in 5.6 is similar to equation in 5.4. Thus,

$$y = \frac{f(x)}{h(x)} \quad (5.7)$$

is solution of equation in 5.6,

where *h(x)* is a function that satisfies equation in 5.4 and *f (x)* is a function that satisfies equation in 5.5.

Therefore, y in 5.7 is solution of equation 4.1.

where *h(x)* is a function that satisfies equation in 5.4 and *f (x)* is a function that satisfies equation in 5.5.

We note that undetermined functions of equation in 4.1 are *h(x)* that satisfies equation in 5.4 and *f(x)* that satisfies equation in 5.5.

We can determine *h(x)* in equation 5.4 because equation in 5.4 is separable first order ordinary differential equation in *h(x).* Also, we can determine *f(x)* in equation 5.5 because equation in 5.5 is separable first order ordinary differential equation in *f(x)* by replacing determined *h(x)* in 5.5.

Result and Discussion

Most authors of differential equations used integrating factor to derive solution method for solving first order ordinary differential equations. In this manuscript, we derived direct method for solving linear first order ordinary differential equations without depending on integrating factor. Moreover, we formed two undetermined functions to derive solution method for solving linear first order ordinary differential equations.

Conclusion

In this manuscript, we formed two undetermined functions *h(x)* in equation 5.4 and *f(x)* in equation 5.5 to derive solution method of equation in 4.1. Finally, we found solution of 4.1. That is,

$$y = \frac{f(x)}{h(x)}$$

is solution of 4.1, where *h(x)* is a function that satisfies equation in 5.4 and *f (x)* is a function that satisfies equation in 5.5.

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