Wavelet-Based Beta Estimation: Applications to Indian Stock Market

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This paper applies the multi-scale beta estimation approach based on wavelet analysis to all stocks comprising BSE-Sensex. Betas are calculated based on the wavelet decomposition from the Maximal overlap discrete wavelet transform (DWT). It is shown that the multi-scale beta estimation approach is useful in certain cases.

JEL classification: G12, C49

Key Words: Beta, Wavelet

1. Introduction

The assessment of market risk has posed a great challenge to financial pundits and academic researchers (see, for instance, Granger, 2002 for an overview). Market risk arises from the random unanticipated changes in the prices of financial assets and measuring it is crucial for investors. Although the measurement of market risk has a long tradition in finance, there is still no universally agreed upon definition of risk. The modern theory of portfolio analysis dates back to the pioneering work of Harry Markowitz in the 1950s. The starting point of portfolio theory rests on the assumption that investors choose between portfolios on the basis of their expected return on one hand, and the variance of their return, on the other. Later on, the Capital Asset Pricing Model (CAPM) emerged through the contributions of Sharpe (1964) and Lintner (1965a, 1965b). According to the CAPM, the relevant risk measure in holding a given asset is the systematic risk, since all other risks can be diversified away through portfolio diversification.

Several studies raised the issue that beta estimation differs depending on the interval used in calculating return [Fama (1970), Pogue and Solnic (1974), Levhari and Levy(1977), Smith (1978), and Hawawini (1980)]. In a seminal work Frankfurter, Leung, and Brockman (1994) use US data to show that the mean and variance of beta increases as return interval increases

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from daily to yearly time horizons. Bjornson, Kim, and Lee (1999) suggest that beta reflects macroeconomic risks that may have different frequencies (high or low) and thus the sensitivity of beta to different time intervals may reflect the impact of those risks. In brief, empirical observations reveal that beta changes, as investment horizons are changed. Traditional methodology of estimating beta therefore will be inappropriate statistically, since lot of information will be lost about the beta dynamics across different intervals. Basically, a security market consists of thousands of traders and investors with different time horizons in their minds regarding their investment decisions. Owing to the different decision-making time horizons among investors, the true dynamics of the relationship between stock returns and risk estimates are likely to vary depending on the time horizon of the investors. Therefore, their perception and measurement of risk are not presumed to be the same. Financial analysts have long recognized the need to incorporate different time scales in line with the investor's decision making in the financial markets. However, due to the lack of appropriate analytical tools to decompose data into more than two time scales, the analysis was restricted, until recently, to two time scales i.e., short-run and long-run only (In & Kim, 2006). A relatively new approach known as wavelet analysis that takes care of different time scales or horizons in decision making would hopefully address that gap. In this paper, we therefore, re-examine risk measurement through a novel approach, wavelet analysis. Wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis in the sense that both time and frequency domains are taken into account.

The paper organizes as follows. Section 2 briefs about the wavelet technique, describes the methodology and provides the data and its description used in this study. Section 3 reports the results with inferences and the final section concludes.

2. Data and Methodology

The article uses daily data of all the stock comprising BSE-30 for a period of 28 months from 5 January 2010 to 31st march 2012. The basic motivation to consider BSE-30 is due to the fact that BSE-30 is a representative index of Indian stock market comprising largest thirty capitalised companies. For each sample company, daily return series (562 observations) have been collected. In addition daily market indices for similar period of time also have been collected. Data has been collected from official website of Bombay Stock Exchange (BSE). After calculating the return series for every stock and the market, wavelet analysis is used to separate out each return series into its constituent multi resolution (multi-horizon) components. For this purpose *Maximal overlap discrete wavelet transform (DWT)* has been applied to obtain a multistage decomposition of the return series at different scale crystals (j) as follows: D1 (2–4 days), D2 (4–8 days) days, D3 (8–16 days), D4 (16–32 days), D5 (32–64 days) and S5 (64-128 days).

The wavelet methodology is used to decompose the market returns and company returns in to different time scales. Wavelet are similar to a sine and cosine functions because they oscillate around zero, but differ because they are well localized both in the time and frequency domains. In contrast to Fourier analysis wavelets are compactly supported, as all projections of a signal onto the wavelet space are essentially local, not global, and thus need not be homogeneous over time. Wavelets are flexible in handling variety of non-stationary signals. Wavelets, in opposition to time and frequency domain analyses, consider non-stationarity an intrinsic property of the data rather than a problem to be solved by pre-processing the data. There are two basic wavelet functions: the father wavelet and the mother wavelet. Formally the father wavelets can be represented as

$$\Phi_{j,k} = 2^{-j/2}\Phi\left(\frac{t-2^{j}k}{2^{j}}\right) \qquad (1)$$

Defined as non-zero over a finite time length support that corresponds to mother wavelets given by

$$\psi_{j.k} = 2^{-j/2}\psi\left(\frac{t-2^{j}k}{2^{j}}\right) \qquad (2)$$

Where J=1,..., J in a J-level decomposition. The father wavelet integrates to one and reconstructs the trend component (longest time scale component) of the series. The mother wavelet integrates to zero and describes all deviations from the trend. In order to compute the decomposition, wavelet coefficients at all scales representing the projections of the time series onto the basis generated by the chosen family of wavelets need to be calculated first, they are

$$d_{j,k} = \int f(t)\psi_{j,k}$$

$$S_{j,k} = \int f(t)\Phi_{j,k}$$

Coefficients $d_{j.k}$ and $S_{j,k}$ are wavelet transform coefficients representing the projection onto mother and father wavelets respectively.

The series or function $f(t)$ in $L^2(R)$ can be shown in wavelet representation as

$$f(t)=\sum_k S_{j,k}\Phi_{j,k}(t)+\sum_k d_{j,k}\psi_{j,k}(t)+\cdots+\sum_k d_{j,k}\psi_{j,k}+\cdots+\sum_k d_{1,k}\psi_{1,k}(t) \quad (3)$$

Here J refers to the number of scales (multiresolution components) and K ranges from 1 to the number of coefficients in the specified components. The original time series f_t in multiresolution decomposition framework can be given as

$$f(t)=S_j+D_j+D_{j-1}+\cdots D_j+\cdots+D_1 \quad (4)$$

S_j , D_j represent $\sum_k S_{j,k}\Phi_{j,k}(t)$ and $\sum_k d_{j,k}\psi_{j,k}(t)$ respectively with $j=1,\cdots,J$ .The sequential set of terms ($S_j, D_j,...D_j,...,D_1$) in equation (4) show the components of original unfiltered series represented at different resolutions.

Following Yamanda (2006) we estimate the following two equations:

$$R_{i,t}=\alpha+\beta R_{m,t}+\mu_t \quad (5)$$

$$R_{i,t}=\gamma+\beta_s R^s_{m.t}+\beta_l R^l_{m,t}+\varepsilon_t \quad (6)$$

Where, β in equation (5) measures the risk associated with a company stock price whereas, β_s and β_l are coefficients associated with a short periodicity series and a long-periodicity series of market returns.

3. Results and Discussion:

Using equation (5) we estimate conventional Betas and Equation (6) is used to estimate the short-periodicity component and the long-periodicity components of market returns. This is done by decomposing the data into short and long-periodicity series using the maximal overlap discrete wavelet transform (MODWT)[1]. We choose the Maximal Overlap Discrete Wavelet Transform (MODWT) over the more conventional orthogonal DWT because, by

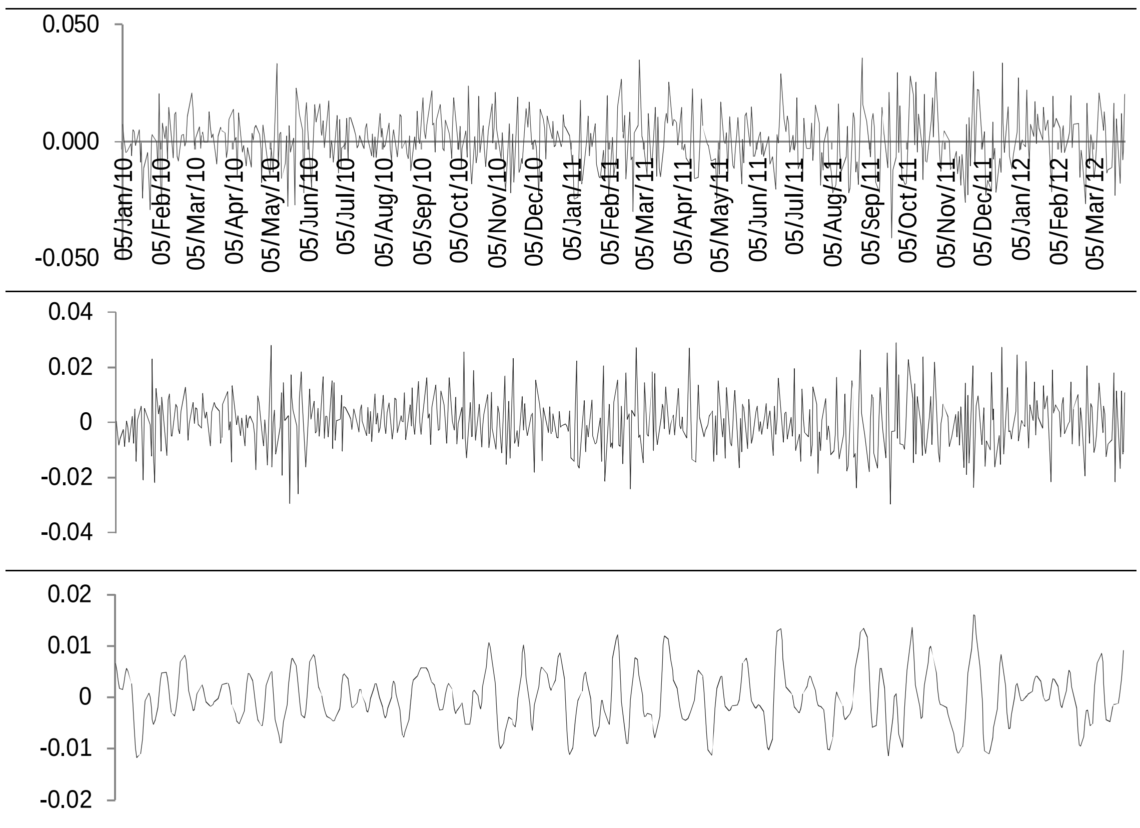
[1] *Short periodicity and long periodicity of market return series are defined as (D1+D2+D3) and (D4+D5+S5) .respectively. Since orthogonality conditions are lost using MODWT we tested the short periodicity and long periodicity components for multicolinearity which was found too low (0.06) to affect the estimation procedure.*

giving up orthogonality, the MODWT gains attributes that are far more desirable in economic applications. For example, the MODWT can handle input data of any length, not just powers of two; it is translation invariant – that is, a shift in the time series results in an equivalent shift in the transform; it also has increased resolution at lower scales since it oversamples data (meaning that more information is captured at each scale); the choice of a particular wavelet filter is not so crucial if MODWT is used and, finally, excepting the last few coefficients, the MODWT is not affected by the arrival of new information.

Table.1 reports the conventional beta estimates and the wavelet-based beta estimates for the 29 stock portfolios. The results for the conventional equation (5) are reported in the first, second and third column. The results for the wavelet-based equation (6) are reported from fourth to seventh column. The last column tabulates the F -values for the null hypothesis *H0:* $\beta_s = \beta_l$[2]. This table indicates that the

Figure 1

Plots from top to bottom represent unfiltered, short periodicity and long periodicity market return series.



[2] *F value is based on wald test statstic*

Table. 1

Conventional and wavelet-based beta estimates

BSE Sensex Co's	α	β	$AdjR^2$	γ	β_s	β_l	$AdjR^2$	F
Cipla	-0.00	0.49	0.14	-0.00	0.51	0.42	0.14	0.37
BHEL	-0.00	1.14	0.12	-0.00	1.06	1.40	0.12	0.70
HDFC	-0.00	0.90	0.07	-0.00	0.86	0.99	0.07	1.88
SBI	-0.00	1.19	0.48	-0.00	1.15	1.32	0.48	1.40
HDFC Bank	-0.00	0.94	0.08	-0.00	0.88	1.12	0.08	3.56***
Hero Motors	0.00	0.54	0.09	0.00	0.50	0.67	0.09	42.95*
Infosys	0.00	0.87	0.41	0.00	0.91	0.74	0.41	3.29***
ONGC	-0.00	0.74	0.05	0.00	0.73	0.76	0.05	0.03
Reliance	-0.00	1.13	0.56	-0.00	1.17	1.02	0.56	1.89
TATA Power	-0.00	0.85	0.05	-0.00	0.79	1.05	0.05	1.41
Hindalco	-0.00	1.58	0.50	-0.00	1.59	1.55	0.50	0.05
TATA Steel	-0.00	1.39	0.53	-0.00	0.08	1.33	0.53	21.64*
L&T	-0.00	1.15	0.49	-0.00	1.12	1.24	0.49	0.90
Mahindra & Mahindra	-0.00	1.06	0.18	-0.00	1.00	1.24	0.18	2.42
TATA Motors	-0.00	1.73	0.22	-0.00	1.69	1.85	0.22	0.85
Hindustan Unilever	0.00	0.43	0.12	0.00	0.43	0.42	0.11	0.00
ITC	0.00	0.57	0.06	0.00	0.61	0.45	0.07	1.18
Sterlite	-0.00	1.35	0.16	-0.00	1.25	1.65	0.16	1.61
Wipro	-0.00	0.81	0.16	-0.00	0.88	0.59	0.16	0.14
Sun Pharama	-0.00	0.59	0.03	-0.00	0.56	0.68	0.03	0.26
GAIL	-0.00	0.55	0.19	-0.00	0.54	0.60	0.18	0.21
ICICI	0.00	1.49	0.66	0.00	1.50	1.45	0.66	0.15
Jindal Steel	-0.00	1.19	0.45	-0.00	1.14	1.33	0.45	1.45
Air Tel	0.00	0.78	0.20	0.00	0.87	0.48	0.21	6.14**
Maruti Suzuki	-0.00	0.72	0.21	-0.00	0.69	0.80	0.21	0.57
TCS	0.00	0.89	0.35	0.00	0.90	0.85	0.35	0.13
NTPC	-0.00	0.71	0.34	-0.00	0.71	0.75	0.34	0.10
DLF	-0.00	1.52	0.46	-0.00	1.42	1.80	0.46	3.63**
Bajaj Auto	0.00	0.69	0.08	0.00	0.65	0.82	0.08	0.36
Average	-0.00	0.96	0.88	-0.00	0.95	1.01	0.88	4.36**

****, **,* indicate significant f Statistic at10%, 5% and 1% respectively.*

conventional beta estimates are between the wavelet-based beta estimates and are approximately the average of them. Second, the null hypothesis, *H0:* $\beta_s = \beta_l$ is not rejected for 23 of the 29 companies at the 10% level of significance. Third, the null hypothesis is rejected for companies such as HDFC Bank, Hero Motors, Infosys, Airtel and combined average returns. To be precise, for HDFC Bank, Hero Motors and average portfolios $\beta_s < \beta_l$

whereas for Infosys, Airtel and DLF $\beta_s > \beta_l$. These results imply that returns on securities in HDFC Bank, Hero Motors, Airtel and from combined 30 stock portfolio are less volatile, in the short-run than conventional considerations suggest. These empirical results indicate that while conventional beta estimates are useful in most cases, in some cases the wavelet based beta estimates are useful in understanding the sensitivity of the returns of particular stocks to the returns on the market index.

Table.2

Multi-Scale Wavelet Beta Estimates on each of the Recomposed Stocks

BSE Sensex Companies	D1	D2	D3	D4	D5	S5
CIPLA	*0.52*	*0.47*	*0.52*	*0.33*	*0.65*	*0.43*
	(0.15)	***(0.14)***	***(0.17)***	***(0.08)***	***(0.33)***	***(0.13)***
BHEL	*0.80*	*1.70*	*1.42*	*1.09*	*1.02*	*0.70*
	(0.06)	***(0.25)***	***(0.18)***	***(0.12)***	***(0.09)***	***(0.05)***
HDFC	0.91	0.90	0.71	1.09	1.49	0.12
	(0.07)	***(0.09)***	***(0.06)***	***(0.12)***	***(0.18)***	***(0.00)***
STATE BANK OF INDIA	1.10	1.19	1.27	1.47	0.46	1.63
	(0.48)	***(0.45)***	***(0.47)***	***(0.58)***	***(0.03)***	***(0.62)***
HDFC Bank	0.91	0.69	1.07	1.62	0.58	1.67
	(0.07)	***(0.05)***	***(0.13)***	***(0.23)***	***(0.03)***	***(0.27)***
Honda Motors	0.56	0.44	0.58	0.57	1.10	0.50
	(0.10)	***(0.05)***	***(0.16)***	***(0.09)***	***(0.34)***	***(0.10)***
Infosys	0.89	0.92	0.77	0.80	0.81	0.80
	(0.45)	***(0.40)***	***(0.36)***	***(0.34)***	***(0.40)***	***(0.42)***
ONGC	0.79	0.78	1.02	0.84	-0.03	1.15
	(0.06)	***(0.06)***	***(0.06)***	***(0.07)***	***(-0.00)***	***(0.13)***
Reliance	1.14	1.24	1.09	0.83	1.29	0.95
	(0.56)	***(0.58)***	***(0.55)***	***(0.47)***	***(0.65)***	***(0.58)***
TATA Power	1.09	0.67	0.60	1.34	1.88	1.03
	(0.08)	***(0.03)***	***(0.03)***	***(0.16)***	***(0.30)***	***(0.08)***
Hindalco	1.59	1.67	1.59	1.58	1.66	1.52
	(0.48)	***(0.54)***	***(0.51)***	***(0.54)***	***(0.65)***	***(0.53)***
TATA Steel	1.50	1.26	1.35	1.32	1.71	1.62
	(0.56)	***(0.50)***	***(0.46)***	***(0.54)***	***(0.70)***	***(0.62)***
LARSEN AND TURBO	1.10	1.18	1.03	1.49	1.36	1.46
	(0.48)	***(0.50)***	***(0.46)***	***(0.62)***	***(0.46)***	***(0.63)***
Mahindra & Mahindra	0.83	0.74	1.05	1.04	1.24	1.44
	(0.10)	***(0.09)***	***(0.21)***	***(0.20)***	***(0.17)***	***(0.03)***
TATA Motors	-1.16	0.47	1.67	1.23	0.95	2.15
	(0.11)	***(0.01)***	***(0.17)***	***(0.15)***	***(0.09)***	***(0.34)***
Hindustan Unilever	-0.30	0.05	0.18	0.34	0.71	0.17
	(0.06)	***(0.00)***	***(0.02)***	***(0.06)***	***(0.27)***	***(0.02)***
ITC	0.70	0.54	0.45	0.49	0.52	0.13

	(0.09)	***(0.06)***	***(0.05)***	***(0.05)***	***(0.07)***	***(0.00)***
Sterlite	1.22	1.34	1.73	1.55	1.78	1.11
	(0.13)	***(0.16)***	***(0.24)***	***(0.21)***	***(0.23)***	***(0.11)***
Wipro	0.9	0.88	0.61	0.68	0.92	0.52
	(0.18)	***(0.18)***	***(0.13)***	***(0.16)***	***(0.16)***	***(0.08)***
Sun Pharama	0.39	0.87	0.64	0.34	1.32	1.00
	(0.01)	***(0.07)***	***(0.05)***	***(0.01)***	***(0.12)***	***(0.12)***
GAIL	0.61	0.41	0.63	0.58	0.58	0.40
	(0.20)	***(0.11)***	***(0.23)***	***(0.26)***	***(0.39)***	***(0.23)***
ICICI	0.22	0.56	1.54	1.46	1.05	1.65
	0.03	***0.04***	***0.70***	***0.75***	***0.70***	***0.81***
Jindal Steel	1.13	1.22	1.31	1.22	1.46	1.30
	(0.41)	***(0.43)***	***(0.55)***	***(0.49)***	***(0.64)***	***(0.71)***
Air Tel	0.91	0.74	0.50	0.45	0.82	0.71
	(0.26)	***(0.15)***	***(0.11)***	***(0.08)***	***(0.28)***	***(0.29)***
Maruti Suzuki	0.66	0.76	0.68	0.93	0.62	1.33
	(0.18)	***(0.23)***	***(0.22)***	***(0.30)***	***(0.21)***	***(0.55)***
TCS	0.91	0.89	0.90	0.98	0.89	0.66
	(0.38)	***(0.34)***	***(0.36)***	***(0.37)***	***(0.38)***	***(0.36)***
NTPC	0.77	0.63	0.70	0.78	0.67	0.80
	(0.34)	***(0.28)***	***(0.41)***	***(0.39)***	***(0.46)***	***(0.55)***
DLF	1.45	1.46	1.59	1.78	1.95	1.66
	(0.46)	***(0.40)***	***(0.54)***	***(0.54)***	***(0.54)***	***(0.69)***
Bajaj Auto	0.66	0.68	1.04	0.57	0.32	0.13
	(0.08)	***(0.07)***	***(0.20)***	***(0.05)***	***(0.02)***	***(0.00)***
	0.96	0.97	1.01	0.99	1.04	0.24
AVERAGE	***(0.87)***	***(0.87)***	***(0.90)***	***(0.90)***	***(0.87)***	***(0.06)***

Values in parenthesis represents Adjusted R^2 & D1 to S6 represents scale-wise Betas of sample companies

In order to show weather the Beta estimation in India is a multi scale phenomena we decompose both market returns and company returns into six scales using (MODWT). Table. 2 reports the Multi scale beta estimation results of equation (5). The beta of individual stock is evaluated at all six scales. Since we employ daily data in our analysis, wavelet scales are such that scale one corresponds to the period of 2-4 days, scale 2- 4 to 8 days, Scale 3, 8 to 16 and so on dynamics. Scale S6 is the highest one at which we can assess the beta of each stock and it corresponds to 126-256 day dynamics i.e., approximately one year. Our results demonstrate the multi-scale tendency of the average beta coefficients in Indian stock market. These findings directly contradict with the results of Norsworthy et al. (2000), Fernandez (2006), Maish (2010) who observed the behaviour of R^2 decreased monotonically when moved to higher scales (longer intervals). In other words their results conclude that market returns are more able to explain individual stock return at lower scales (shorter intervals) than higher scales. However, the present piece of work finds interesting results. Although the beta estimates turn significant at each level, the higher R^2 associated with scale-3, scale-4, and scale-5 respectively implies that major part of market portfolios influence on individual stocks is at medium to higher frequencies. The findings indicate that investor with medium and longer investment horizon has to respond to every fluctuation in realized returns.

4. Conclusion:

Using wavelets we examined the dynamics of stock returns of 29 firms from Indian Stock market. We calculated two betas based on the short periodicity and long periodicity of market returns based on (MODWT). We have shown that the conventional beta estimate is an "average" of the wavelet-based beta estimates for most of the cases. This means that the conventional beta estimates are useful in most cases, but in some cases, betas for short periodicity and long periodicity market returns were significantly different suggesting wavelet-based beta estimates are useful for understanding the sensitivity of the returns of particular securities to the returns on the market index. Further we estimated multi-scale betas on each stock comprising BSE-sensex and observed that beta of all stocks is not stable over time, due to multi-trading strategies of investors. In particular we found market risk were concentrated by and large, at the medium and higher frequencies of data. In other words the findings imply that predictions of the Market model would be more relevant at medium- to long-run horizons as compared to short time horizons.

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