A Grill between Weak Forms of Faint Continuity

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Abstract

We are interested introducing new classes of faint continuity namely faint G-γ continuity, faint G-α-continuity and faint G-semi-continuity to obtain their characterizations and some of their properties. AMS Mathematics Subject Classification: (2000) 54D10, 54C10.

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Introduction

The idea of grill topological space aimed at generalize the concept of topological space since it generates newtopology τγ which helps to measure the things that was di cult to measure. Choquet [1] in 1947 was the rst author introduced the idea of grill. It has been explored that there is some of similarity between Choquets concept and that ideals, nets and lters. In 2007, Roy and Mukherjee introduced the def nition of τγ which is related to two operators Φ and Ψ. They have determined the relation between τ and τγ. A number of theories and characterization has been handled in previous studies [2-4] whether respect to sets or functions. Hatir and Jafari [5] have the ability determine the def nition of G-preopen) sets of (Y,τ,Г) is denoted by GS(Y) (resp. ГO(Y), ГO(Y), GPO(Y)).

The family of all G-semi-open (resp. G-α-open, G-γ-open, G-preopen) sets of (Y,τ,Г) containing a point y∈Y is denoted by GS(Y; y) (resp. ГO(Y; y), ГO(Y; y), GPO(Y; y)).

Definition 2.3: "The intersection of all Г-γ-closed (resp. Г-α-closed, Г-semi-closed) sets contained S⊂Y is called Г-γ-closure (resp. Г-α-closure, Г-semi-closure) of S and is denoted by ГCl(S) (resp. ГCl(S), ГCl(S)) [7]."

Definition 2.4: "A function h: (Y;τ,G) → (Z;σ) is said to be Г-γ-continuous at each point y∈Y if for each open set B of Z containing h(y), there exists A∈ГO(Y; y) such that h(A)⊂B. If h has this property at each point of y, then it is said to be Г-γ-continuous [8]."

Definition 2.5: "A subset A of Y is said to be Г-open [9] if for each y∈A there exists an open set U such that h(A) is open in Y for every Г-open set U of Z containing h(y)."

Definition 2.6: The Г-γ-closure of A, denoted by ГCl(A), is defined to be the set of all y∈Y such that for each y∈A there exists an open set U such that A∩ГCl(U) isГ for every Г-open neighborhood U of Y. If A=ГCl(A), then A is called Г-closed. The complement of a Г-γ-open set is called Г-γ-open. It follows that the collection of Г-open sets in a topological space (Y;τ,G) forms a Г-topology τγ on Y. The Г-interior of a Г is def ned by the union of all Г-open sets contained in Г and is denoted by ГInt(A) [10]."

Definition 2.7: "A function h: (Y;τ,G)→(Z;σ) is said to be faintly continuous [9] if h' (H) is open in Y for each Г-open set H of Z'."

Definition 2.8: "A function h: (Y;τ,G)→(Z;σ) is said to be faintly Г-precontinuous [6] at a point y∈Y if for each Г-preopen set B of Z"
containing \( h(y) \), there exists \( A \in \text{GPO}(Y; y) \) such that \( h(A) \subset B \). If \( h \) has this property at each point of \( Y \), then it is said to be faintly \( G\)-precontinuous”.

**Some Fundamental Properties**

In this section, we define some modern classes of functions called faint \( G\)-\( \gamma \)-continuity faint \( G\)-\( \gamma \)-continuity and faint \( G\)-semi-continuity. Depictions and essential properties of these concepts are studied.

**Definition 3.1:** A function \( h: (Y; \tau;G) \rightarrow (Z;\sigma) \) is said to be faintly \( G\)-\( \gamma \)-continuous (resp. faintly \( G\)-\( \alpha \)-continuous, faintly \( G\)-semi-continuity) if for each \( y \in Y \) and each \( \theta \)-open set \( B \) of \( Z \) containing \( h(y) \), there exists \( A \in G\_O(Y; y) \) (resp. \( A \in G\_aO(Y; y) \), \( U \in G\_SO(Y; y) \)) such that \( h(A) \subset B \).

**Theorem 3.2**

For a function \( h: (Y; \tau;G) \rightarrow (Z;\sigma) \), the following statements are equivalent:

1. \( h \) is faintly \( G\)-\( \gamma \)-continuous;
2. \( h^{-1}(H) \) is \( G\)-\( \gamma \)-open in \( Y \) for every \( \theta \)-open set \( H \) of \( Z \);
3. \( h^{-1}(F) \) is \( G\)-\( \gamma \)-closed in \( Y \) for every \( \theta \)-closed set \( F \) of \( Z \);
4. \( h: (Y; \tau;G) \rightarrow (Z;\sigma) \) is \( G\)-\( \gamma \)-continuous;
5. \( \gamma \text{Cl}_G(h^{-1}(W)) \subset h^{-1}(\text{Cl}_G(W)) \) for every subset \( W \) of \( Z \);
6. \( h^{-1}(\text{Int}_\theta(W)) \subset \text{Int}_\theta(h^{-1}(W)) \) for every subset \( W \) of \( Z \).

**Proof:** The proof is similar to the proof of Theorem 3.2.

**Theorem 3.5**

Every \( G\)-\( \gamma \)-continuous function is faintly \( G\)-continuous.

**Proof:** It is clear from Definitions 3.1 and Theorem 3.2.

The converse of Theorem 3.5 is not true in general as it can be seen in the following example.

**Example 3.6:** Let \( Y=[1; 2; 3; 4], \tau=[Y,\phi,\{1\},\{3\},\{1,3\}] \) and \( G=[Y, \{2,1\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{2,3,4\}] \). The function \( h: (Y, \tau;G) \rightarrow (Y, \sigma) \) where \( h(1)=2, h(2)=3, h(3)=4, h(4)=1 \) is faintly \( G\)-\( \gamma \)-continuous function but not \( G\)-\( \gamma \)-continuous function.

**Corollary 3.7:** Each faintly continuous function is faintly \( G\)-\( \gamma \)-continuous function (Figure 1).

**Remark 3.8**

1. (1) The following figure shows the relationship among these new classes of functions and other corresponding types.
2. (2) the converse are not true in general as shown by the following examples.

**Example 3.9:** Let \( Y=Z=[1; 2; 3], \tau=[Y,\phi,\{1\},\{2\},\{1,2\}], \sigma=\{Z,\phi,\{1\},\{2\},\{1,2\},\{1,3\}\} \) and \( G=P(Y)\{\phi\} \). The identity function \( h: (Y, \tau;G) \rightarrow (Z, \sigma) \) is faintly \( G\)-\( \gamma \)-continuous function but not faintly \( G\)-\( \gamma \)-precontinuous function since \( h^{-1}(\{1,3\}) \text{GPO}(Y, \tau) \) where \( \{1,3\} \subset \sigma \).

**Example 3.10:** Let \( Y=Z=[1; 2; 3], \tau=[Y,\phi,\{1\},\{2\},\{1,2\}], \sigma=\{Z,\phi,\{1\},\{2\},\{1,2\},\{1,3\}\} \) and \( G=P(Y)\{\phi\} \). The identity function \( h: (Y, \tau;G) \rightarrow (Y, \sigma) \) is faintly \( G\)-\( \gamma \)-continuous function but not \( G\)-semi-continuous function.

**Example 3.11:** Let \( Y=Z=[a, b, c], \tau=[Y,\phi,\{a\},\{b\},\{a, b\}], \sigma=\{Z,\phi,\{a\},\{b\},\{a, b\}\} \).
Let Z=\{1, 2, 3\}; \tau=\{Y, \emptyset, \{1, 2, 3\}, \emptyset\}, σ=\{Z, \emptyset, \{1, 2\}, \{1, 3\}\} and G=P(Y)=[\emptyset, \emptyset, \emptyset]. The function h: (Y, τ, G) → (Y, σ) where h(a)=h(b)=a, h(b)=b is faintly G-continuous function but not faintly G-α-continuous function.

**Example 3.12:** Let Z=\{1, 2, 3\}, \tau=\{Y, \emptyset, \{1, 2\}, \{1, 3\}\}, and σ=\{Z, \emptyset, \{1, 2\}, \{1, 3\}\} and G=P(Y)=[\emptyset, \emptyset, \emptyset]. The function h: (Y, τ, G) → (Y, σ) where h(1)=h(3)=1, h(2)=2 is faintly G-continuous function but not G-continuous function.

**Definition 3.13:** A function h: (Y, τ) → (Z, σ) is named quasi-θ-continuous if h^{-1}(H) is G-θ-continuous and g: (Y, τ, G) → (Z, σ) is quasi-θ-continuous, then g ° h: (X, τ, G) → (Z, σ) is faintly G-θ-continuous function.

**Proof:** Let W be a θ-open set of (Z, σ). Then \( g^{-1}(W) = \emptyset \)-open in (Y, τ) θ-open set H of (Z, σ).

**Theorem 3.14**

If the function h: (X, τ, G) is faintly G-θ-continuous and g: (Y, τ, G) → (Z, σ) is quasi-θ-continuous, then g ° h: (X, τ, G) → (Z, σ) is faintly G-θ-continuous function.

**Definition 3.16:** A space (Y, τ, G) is named G-θ-regular if Y cannot be written as a union of two nonempty disjoint G-θ-open sets.

**Theorem 3.17**

Let h: (Y, τ, G) → (Z, σ) be a faintly G-θ-continuous surjection and (Z, σ) is a regular space, then h is G-θ-continuous function.

**Proof:** Let H be an open set of Z. Since Z is regular, H is θ-open in Z and h is faintly G-θ-continuous, then by Theorem 3.2, we have h^{-1}(H) is G-θ-open and hence h is G-θ-continuous function.

**Definition 3.16:** A space (Y, τ, G) is named G-θ-connected if Y cannot be written as a union of two nonempty disjoint G-θ-open sets.

**Theorem 3.17:**

Let h: (Y, τ, G) → (Z, σ) be a faintly G-θ-continuous surjection and (Y, τ, G) is a G-θ-connected space, then Z is a connected space.

**Proof:** Suppose that (Z, σ) is not connected. Then there exist two nonempty open sets H and H, \( (Y, σ) \) such that \( H \cap H = \emptyset \) and \( H \cup H = Z \). Therefore, by Theorem 3.2, we have \( h^{-1}(H) \cap h^{-1}(H) = \emptyset \) and \( h^{-1}(H) \cup h^{-1}(H) = Z \). Since Z is regular, \( h^{-1}(H) \) and \( h^{-1}(H) \) are nonempty subsets of Z. Since Y is θ-open, \( h^{-1}(H) \cap h^{-1}(H) = \emptyset \). Because of h is faintly G-θ-continuous, \( h^{-1}(H) \cap h^{-1}(H) \) is G-θ-open. So, (Y, τ, G) is not G-θ-connected. This is a contradiction. (Z, σ) is connected.

**Definition 3.18:** A collection \( \{G_i; \mathbb{E}P\} \) is called a G-θ-open cover of a subset A of a space Y which carries topology \( τ \) and carries G-θ-open cover with grill G if A \( \subset \{G_i; Y_i, G_i \in G_i(O)(y), \mathbb{E}P\} \).

**Theorem 3.19:** Let Y be a space which carries topology \( τ \) with grill G, then it is said to be:

1. G-θ-T₁, (resp. θ-T₁, [11]) if for each pair of different points \( y \) and \( z \) of Y, there exists G-θ-open (resp. θ-open) sets A and B containing \( y \) and \( z \), respectively such that \( z \neq A \) and \( A \neq B \).
2. G-θ-T₂ (resp. θ-T₂, [11]) if for each pair of different points \( y \) and \( z \) of Y, there exists disjoint G-θ-open (resp. θ-open) sets A and B in Y such that \( y \in A \) and \( z \in B \).

**Theorem 3.20**

Let h: (Y, τ, G) → (Z, σ) be faintly G-θ-continuous injection and Z be a θ-T₂-space, then Y is a G-θ-T₂-space.

**Proof:** Let Z be a θ-T₂-space. For any two different points \( y \) and \( z \) in Y, then there exist A; W ∈ σ such that h(\( y \)) ∈ A, h(\( z \)) \( \neq A \), h(y) \( \neq W \) and h(z) \( \in W \) for Z is a θ-T₁-space. Because of h is faintly G-θ-continuous and h(y) \( \neq \) and h(z) \( \neq W \) are G-θ-open subsets of (Y, τ, G) such that h^{-1}(A) and h^{-1}(W) containing y and z, respectively \( z \neq h^{-1}(A) \) and \( y \neq h^{-1}(W) \). Then Y is G-θ-T₂.

**Theorem 3.21**

Let h: (Y, τ, G) → (Z, σ) be a faintly G-θ-continuous-injection function and Z be a θ-T₂-space, then Y is a G-θ-T₂-space.

**Proof:** For any two several points y and z of Y and since Z is a θ-T₂-space, then there exists h(y) \( \neq A \) and h(z) \( \neq B \) for disjoint θ-open sets A and B in Z. h^{-1}(A) and h^{-1}(B) are G-θ-open subsets of (Y, τ, G) such that h^{-1}(A) and h^{-1}(B) containing y and z, respectively by h is faintly G-θ-continuous-injection. Therefore, h^{-1}(A)∩h^{-1}(B)=\emptyset \\text{for ANB=\emptyset}. Then Y is G-θ-T₂.

**Theorem 3.22**

A function h: (Y, τ, G) → (Z, σ) is faintly G-θ-continuous-injection if the graph function g: Y → Y \times Z is faintly G-θ-continuous.

**Proof:** Let y ∈ Y and H be a θ-open set of Z containing h(x). Then \( \exists X \in \emptyset \)-open in Y \times Z. \[6, \text{Theorem 5}\] contains g(y)=(y, h(y)). Therefore, there exists A \( \in G\left(O_y, Y\right) \) such that g(A) \( \subset \emptyset \times H \). This implies that h(A) \( \subset \emptyset \). Thus, h is faintly G-θ-continuous.

**Definition 3.23:** A graph g(h) of a function h: (Y, τ, G) → (Z, σ) called θ-G-closed if for every \( (y; z) \in \emptyset \times Z \), \( g(h)(y; z) \) exists A \( \in G\left(O_y, Y\right) \) and B \( \in \emptyset \), containing such that \( h(A) \cap B = \emptyset \).

**Lemma 3.24:** A graph g(h) of a function h: (Y, τ, G) → (Z, σ) is said to be θ-G-closed in Y \times Z if only if for each \( (y; z) \in \emptyset \times Z \), \( g(h) \) exists A \( \in G\left(O_y, Y\right) \) and B \( \in \emptyset \), containing such that \( h(A) \cap B = \emptyset \).

**Proof:** It is clear from Definition 3.24.

**Theorem 3.25**

If h: (Y, τ, G) → (Z, σ) is faintly G-θ-continuous function and (Z, σ) is regular space, then the following statements are satisfied:

1. Faintly G-θ-continuous function coincide with G-θ-continuous function.
2. Faintly G-α-continuous function coincide with G-α-continuous function.

**Proof:** Let B be any open set in Z. Since Z is regular, B is θ-open in Z. Because of h is faintly G-θ-continuous function, by Theorem 3.2, we have h^{-1}(B) is G-open and h is G-θ-continuous function.
Proving (2, 3) are Similar to (1).

Conclusion

The study of faintly grill topological spaces is very important. It is generalization of faintly topological spaces. So we introduced neoteric classes of functions called faintly $G$-$\gamma$-continuous, faint $G$-$\alpha$-continuous and faint $G$-semi-continuous in grill topological spaces that helps us in many applications such as computer and information systems. Furthermore relationships between different classes are introduced. Also, some of their basic properties of different types of functions between grill topological spaces are obtained.

References