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On the Thermal Conductivity of Magnets near the Point of Phase Transition

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Abstract

An estimate of the thermal conductivity coefficient of ferromagnetic κ at temperatures lying near the point of phase transition, i.e. at temperature $T = T_c - \varepsilon$, where $\varepsilon \ll T_c$. made. The calculations are based on the quasi-classical kinetic equation method, and it shows that $\kappa = \kappa_0 \left(1 - \frac{T}{T_c} + \delta_0\right)^{-\frac{5}{2}}$ with a critical index equal $\frac{5}{2}$, and $\delta_0 = \frac{H_a}{H_E}$, where $H_a -$ field of magnetic anisotropy, $H_E -$ exchange field.

Keywords: Magnetic phase temperature; Quasi-classical kinetic equation; Thermal conductivity factor.

Introduction

When carefully acquainted with a large number of literary sources dedicated to the description of the physical properties of magnetic substances at temperatures in the area of magnetic phase transition [1-10], despite the nearly century-long history of this issue, we have not found an analytical calculation of the temperature dependence of the ferromagnetic thermal conductivity coefficient $\kappa(T)$ at temperature $T = T_c \pm \varepsilon$, where $T_c -$ is Curie's temperature, and $\varepsilon \ll T_c$. Although the general theory of phase transitions, both macroscopic (ref. [11] – [13]) and microscopic (see [14] – [18]) long been constructed, some questions for some reasons have been overlooked and not covered in the literature. That is why the purpose of this communication is to close this small gap associated with calculating of thermal conductivity in the area of magnetic phase temperature. Experimental measurements devoted to this subject , which is typical of any other physical characteristics in the field) indicate a strong growth of thermal conductivity $\kappa_{\max} = \kappa(T_c)$ at $T = T_c$ the value of which can be calculated analytically, and which, as we shall see, has an end value. Looking ahead, we note that the value of this maximum is determined by the value of the $\kappa_{\max} = \kappa_0 \left(\frac{H_E}{H_a}\right)^{\frac{3}{2}}$, where $\kappa_0 -$ characteristic thermal conductivity coefficient, $H_a -$ field of magnetic anisotropy, $H_E = \frac{J_{ex}}{\mu_e} -$ exchange magnetic field, $J_{ex} -$ energy of exchange interaction, $\mu_e -$ Bore's magneton.

Hamiltonian interactions at $T < T_c$

In the theory of phase transitions concerning the unbalanced properties of the substance, the necessary condition is always to know the appropriate time of relaxation.

Specifically, when it comes to thermal conductivity calculation κ need to have an idea of the relaxation time, which in the case of ferromagnets is due to the interaction between magnon and phonon subsystems.

Since we are talking about a gas-kinetic approach, the basic estimated formula in the isotropic case should look according to the general rules of its receipt.

$$\kappa = \frac{1}{3(2\pi)^3} \int \mathrm{v}^2 \tau_{m-ph}(k) d^3k \,, \tag{1}$$

where $\mathrm{v} = \frac{1}{\hbar}\frac{\partial \varepsilon_k}{\partial k} -$ group speed of the magnons, $\varepsilon_k = J_{ex}(ak)^2 + bk_B T_c \left(1 - \frac{T}{T_c}\right) + \mu_e H_a -$ their dispersion, $\hbar -$ Planck's constant, $k -$ wave vector of magnon, $k_B -$ Boltzmann's constant, $\tau_{m-ph}(k) -$ magnon - phonon time of relaxation.

Everywhere we will use the energy system of units, in which we believe that Boltzmann's constant $k_B = 1$ and for the sake of convenience we will put also as the Planck's constant $\hbar = 1$. This will not lead to confusion in dimensions (see below).

Because the group speed of the magnon is $\mathrm{v} = 2J_{ex}a^2k$, it's always possible to think that $d^3k = 4\pi k^2 dk$, and therefore from (1) we have

$$\kappa = \frac{2J_{ex}^2 a^4}{3\pi^2} \int_{k_0}^{\frac{\pi}{a}} k^4 \tau_{m-ph}(k) dk \,, \tag{2}$$

where the lower limit of integration can only be defined after analysis energy and impulse laws for a particular magnon-phonon interaction.

With this goal we need to introduce the energy of magnon – phonon interaction is easy to record for isotropic case considering the crystal symmetry to be cubic.

We can present the relevant interaction in the only form, as

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$$H_{m-ph} = \gamma \int_V M_i M_k u_{ik} dV \ , \qquad (3)$$

where $\gamma -$ is dimensionless strict constant order of units, $\mathbf{M} = (M_i) -$ is the order parameter, i.e. the magnetic moment of the unite volume of the ferromagnet, $_{ik} = -\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) -$ deformation tensor, $\mathbf{u} = (u_i) -$ is vector of deformation, $V -$ is the body volume.

By the way, it's worth noting that interactions like $H_{\text{int}} = \gamma_{iklmn} \int_V M_i u_{kl} u_{nm} dV$ in crystals with any spatial symmetry it's can't be.

It's connected due to the physical fact that any type of interaction must satisfy the rule of invariance in relation to the time inversion operation, that $H_{\text{int}}(t) = H_{\text{int}}(-t)$, where $t -$ is the time.

Because the vector of magnetism is an axial vector in relation to the replacement of inversion time $t \to -t$ i.e. $\mathbf{M}(-t) = -\mathbf{M}(t)$, this indicates that this type of interaction is not possible, although it should be noted that this type of interaction is not prohibited by the laws of energy and momentum.

To rewrite the interaction (3) in the language of operators of birth and destruction of magnons and phonons, it must be presented in a secondary – quantization form. To do this, we should remember that in accordance with Holstein-Primakov's transformations, we have the right to write down that

$$\begin{cases} \mathrm{M}^+ = \mathrm{M}_x + i\mathrm{M}_y \approx M_0\sqrt{2S}a(\mathbf{r}), \\ \mathrm{M}^- = \mathrm{M}_x - i\mathrm{M}_y \approx M_0\sqrt{2S}a^+(\mathbf{r}) \end{cases}. \qquad (4)$$

where $\mathrm{M}_0 = \frac{\mu_e}{a^3}\sqrt{1 - \frac{T}{T_c} + \delta_0} -$ is spontaneous magnetism in the Curie's temperature area, $S -$ is the spin of atom, $a^+(a) -$ is the operator of the birth (destruction) of the magnon at the local point with a radius - vector **r**. The expression for the dimensionless value δ_0 are given below.

In the $\mathbf{k} -$ presentation the operators of berth (destruction) of magnon we can presented as a Fourier's row

$$\begin{cases} a(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} a_k e^{i\mathbf{kr}}, \\ a^+(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} a_{\mathbf{k}}^+ e^{-i\mathbf{kr}}, \end{cases} \qquad (5)$$

where $N -$ full number of atoms in the magnet.

The vector of deformation is also easily carried out with the help of a simple Fourier - decomposition

$$\mathbf{u} = \sum_{\mathbf{q},\alpha} \mathbf{e}_\alpha \sqrt{\frac{\hbar}{\rho \omega_\alpha(q) V}} \left(b_{q,\alpha}^+ + b_{-q,\alpha}\right) e^{i\mathbf{qr}} \ , \qquad (6)$$

where $\mathbf{e}_\alpha -$ is an polarization vector of phonon, index *a* is specify to three possible polarization branches: two acoustic (longitudinal and transversal) and one optical, $\rho -$ is the density of ferromagnet, $\mathbf{q} -$ is a wave vector of phonon, $b_{q,\alpha}^+ (b_{q,\alpha}) -$ is an operator of berth (destruction) of phonon with the wave vector **q** and polarization *a*, $\omega_\alpha(q) -$ is phonon dispersion, where index $\alpha = 1,2,3$.

For the acoustical phonons the dispersion is $\omega_{1,2}(q) = \omega_{l,t}(q) = c_{l,t} q$, where $c_{l,t} -$ is accordingly the longitudinal and transversal speed of the sound. Single designation will be used everywhere below c_s . For optical phonons, we will believe that $\omega_3 = \omega_{opt}(q) = \omega_0 - \Delta(q) \approx \omega_0$.

According to (6) the components of the tensor deformations can then be presented as a form of

$$u_{ik} = \frac{i}{2} \sum_{\mathbf{q},\alpha} (e_{i\alpha} q_k + e_{k\alpha} q_i) \sqrt{\frac{\hbar}{\rho \omega_\alpha(q) V}} \left(b_{q,\alpha}^+ + b_{-q,\alpha}\right) e^{i\mathbf{qr}} . \qquad (7)$$

After substitution (5) and (7) in definition (3) as a result of simple rule-adjusted changes

$$\int_V e^{i(\mathbf{k}_1 - \mathbf{k}_2 \pm \mathbf{q})\mathbf{r}} dV = V\Delta(\mathbf{k}_1 - \mathbf{k}_2 \pm \mathbf{q}) , \qquad (8)$$

we come to the interaction in the form

$$H_{m-ph} = \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\alpha} \Psi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\alpha) a_1^+ a_2 \left(b_{q,\alpha}^+ + b_{-q,\alpha}\right) \Delta(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}) + c.c. \ , \qquad (9)$$

Where the abbreviation $c.c.$ means a complex - conjugated value and indices 1,2 at birth and destruction operators mean a reduction i.e. $1 \equiv \mathbf{k}_1$, $2 \equiv \mathbf{k}_2$. At the same the amplitude of interaction is defined as

$$\Psi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\alpha) = i\frac{4\gamma S M_0^2 V}{N\sqrt{V}} \sqrt{\frac{\hbar}{\rho \omega_\alpha(q)}} (\mathbf{e}_\alpha \mathbf{q} - e_{\alpha z} q_z) \ . \qquad (10)$$

It is clear that in the absence of interaction the main Hamiltonian can be recorded as

$$H_0 = \frac{1}{N} \sum_{\mathbf{k}} \varepsilon_k a_k^+ a_k + \frac{1}{N} \sum_{\mathbf{q},\alpha} \omega_\alpha(q) b_q^+ b_q \ ,$$

where ε_k and $\omega_\alpha(q)$ are determined above.

Calculating the relaxation time in the field of phase transition temperature

We can use the quasi-classical kinetic equation (see, for example, refs. [26] – [29]) in the form

$$\frac{\partial n_k}{\partial t} = L\{n_k, \bar{f}_q\} \ , \qquad (11)$$

where $n_k -$ is an collision integral, $n_k -$ function distribution of magnons, $\bar{f}_q -$ is an equilibrium distribution function of the phonons. The phonons we are assuming an thermostat. At high temperatures in the region of Curie's temperature it can presented in following

$$\bar{f}_q = \frac{1}{e^{\frac{\omega_{q,\alpha}}{T_c}} - 1} \approx \frac{T_c}{\omega_{q,\alpha}} \ . \qquad (12)$$

Before moving on to specific calculations, it is worth noting that the equation (11) will be fairly, if the condition is met $\varepsilon_k \gg \frac{\hbar}{\tau_{m-ph}(k)}$, where $\tau_{m-ph}(k) -$ is the magnon – phonon relaxation time.

As we shall see later (see formula (20)), this condition is fulfilled near the phase point and it is quite possible to use the equation (11).

According to the interaction (9) the integral of collisions will look like this

$$L\{n_k\} = 2\pi \sum_{\mathbf{k}_1,\mathbf{q},\alpha} |\Psi|^2 \left\{(1+n_k)n_1\left(1+\bar{f}_{q,\alpha}\right) - n_k(1+n_1)\bar{f}_{q,\alpha}\right\}\Delta(\mathbf{k}-\mathbf{k}_1+\mathbf{q})\delta\left(\varepsilon_k-\varepsilon_1+\omega_{q,\alpha}\right)+$$
$$+2\pi \sum_{\mathbf{k}_1,\mathbf{q},\alpha} |\Psi|^2 \left\{(1+n_k)n_1\bar{f}_{q,\alpha} - n_k(1+n_1)\left(1+\bar{f}_{q,\alpha}\right)\right\}\Delta(\mathbf{k}-\mathbf{k}_1-\mathbf{q})\delta\left(\varepsilon_k-\varepsilon_1-\omega_{q,\alpha}\right), \qquad (13)$$

where the presence of delta function automatically takes into account the law of energy conservation. According to (13) the relaxation time in the "tau - approximation" will be

$$\frac{1}{\tau_{m-ph}(k)} = 2\pi \sum_{\mathbf{k}_1,\mathbf{q},\alpha} |\Psi|^2 \left(\bar{f}_{q,\alpha} - \bar{n}_1\right)\Delta(\mathbf{k}-\mathbf{k}_1+\mathbf{q})\delta\left(\varepsilon_k-\varepsilon_1+\omega_{q,\alpha}\right)+$$
$$+2\pi \sum_{\mathbf{k}_1,\mathbf{q},\alpha} |\Psi|^2 \left(1+\bar{f}_{q,\alpha}+\bar{n}_1\right)\Delta(\mathbf{k}-\mathbf{k}_1-\mathbf{q})\delta\left(\varepsilon_k-\varepsilon_1-\omega_{q,\alpha}\right), \qquad (14)$$

where the equilibrium function distribution of magnons at the temperatures near the Curie's point is

$$\bar{n}_k = \frac{1}{e^{\frac{\varepsilon_k}{T_c}}-1} \approx \frac{T_c}{\varepsilon_k}. \qquad (15)$$

As a result, instead of (14) we get

$$\frac{1}{\tau_{m-ph}(k)} \approx 2\pi T_c \varepsilon_k \sum_{\mathbf{k}_1,\mathbf{q},\alpha} \frac{|\Psi|^2}{\omega_{q,\alpha}\left(\varepsilon_k+\omega_{q,\alpha}\right)}\Delta(\mathbf{k}-\mathbf{k}_1+\mathbf{q})\delta\left(\varepsilon_k-\varepsilon_1+\omega_{q,\alpha}\right)+$$
$$+2\pi T_c \varepsilon_k \sum_{\mathbf{k}_1,\mathbf{q},\alpha} \frac{|\Psi|^2}{\omega_{q,\alpha}\left(\varepsilon_k-\omega_{q,\alpha}\right)}\Delta(\mathbf{k}-\mathbf{k}_1-\mathbf{q})\delta\left(\varepsilon_k-\varepsilon_1-\omega_{q,\alpha}\right), \qquad (16)$$

After summing up on $\mathbf{k}_1$ we have of (16)

$$\frac{1}{\tau_{m-ph}(k)} \approx 2\pi T_c \varepsilon_k \left[\sum_{\mathbf{q},\alpha} \frac{|\Psi|^2}{\omega_{q,\alpha}\left(\varepsilon_k+\omega_{q,\alpha}\right)}\delta\left(\varepsilon_k-\varepsilon_{\mathbf{k}+\mathbf{q}}+\omega_{q,\alpha}\right)+\sum_{\mathbf{q},\alpha} \frac{|\Psi|^2}{\omega_{q,\alpha}\left(\varepsilon_k-\omega_{q,\alpha}\right)}\delta\left(\varepsilon_k-\varepsilon_{\mathbf{k}-\mathbf{q}}-\omega_{q,\alpha}\right)\right] \qquad (17)$$

A simple analysis of energy and momentum laws, taking into account the rules of transition from summation to integration

$$\sum_{\mathbf{q}}(\ldots) = \frac{V}{(2\pi)^3}\int(\ldots)d^3q \qquad (18)$$

allows us to get the following interim expression

$$\frac{1}{\tau_{m-ph}(k)} = F\int_{q_1}^{q_2} \frac{q}{\varepsilon_k^2-c_s^2q^2}\left[1-\left(\frac{q-\frac{c_s}{\beta}}{2k}\right)^2\right]dq + G\int_{\tilde{q}_1}^{\tilde{q}_2} q^3\left[1-\left(\frac{q^2-\frac{\omega_0}{\beta}}{2kq}\right)^2\right]dq - G\int_{\bar{q}_1}^{\bar{q}_2} q^3\left[1-\left(\frac{q^2+\frac{\omega_0}{\beta}}{2kq}\right)^2\right]dq \qquad (19)$$

where the parameters are

$$F = \frac{T_c\varepsilon_k^2}{3\pi\rho\beta k c_s^2\hbar^2}\left(\frac{4\gamma S M_0^2 V}{N}\right)^2,$$
$$G = \frac{T_c\varepsilon_k}{6\pi\rho\beta k\omega_0^3\hbar^3}\left(\frac{4\gamma S M_0^2 V}{N}\right)^2,$$
$$\beta = a^2\omega_E,$$
$$q_{1,2} = \frac{c_s}{\beta} \pm 2k,\ \tilde{q}_{1,2} = \sqrt{k^2+\frac{\omega_0}{\beta}} \pm k,\ \bar{q}_{1,2} = k \pm \sqrt{k^2-\frac{\omega_0}{\beta}}. \qquad (20)$$

In fact, simple but rather cumbersome calculations of the integrals featured in (19) lead to the next final result for the time of relaxation of interest to us

$$\frac{1}{\tau_{m-ph}(k)} = \frac{F}{2c_s^2}\left(1-\frac{c_s^2}{4\beta^2k^2}-\frac{\varepsilon_k^2}{4\hbar^2k^2c_s^2}\right)\ln\left|\frac{c_s^2q_1^2-\varepsilon_k^2}{c_s^2q_2^2-\varepsilon_k^2}\right| - \frac{F}{\beta k c_s} +$$
$$+\frac{F\varepsilon_k}{4\beta\hbar k^2c_s^2}\ln\left|\frac{(\varepsilon_k+c_sq_2)(\varepsilon_k-c_sq_1)}{(\varepsilon_k-c_sq_2)(\varepsilon_k+c_sq_1)}\right| + \frac{4Gk}{3}\left[\left(k^2+\frac{\omega_0}{\beta}\right)^{\frac{3}{2}}-\left(k^2-\frac{\omega_0}{\beta}\right)^{\frac{3}{2}}\right], \qquad (21)$$

where $q_{1,2}$ identified in (20), and the wave vector of the magnon should be "squeezed" in the segment

$$\sqrt{\frac{\omega_0}{\beta}} = k_0 \le k \le \frac{\pi}{a}. \qquad (22)$$

Expression analysis (21) shows that only the large values of the magnon wave vector will make the greatest contribution to the thermal conductivity coefficient. This means that the last term in the expr. (21) will be the most significant. Leaving only it and substituting the result in the general definition (2), we get

$$\kappa \approx \frac{\rho a^5\omega_E^2\hbar\omega_0^2}{\left(4\gamma S a^3 M_0^2\right)^2}\int_{\sqrt{\frac{\omega_0}{\beta}}}^{\frac{\pi}{a}} \frac{dk}{(ak)^2+b\left(1-\frac{T}{T_c}\right)+\frac{H_a}{H_E}}. \qquad (23)$$

Due to the convergence of this integral, we can put the lower limit of integration to zero, and the upper limit - infinity.

Assuming that at a point $T = T_c$ over an extremely short period of time τ, which is significantly less than all relaxation times, an instant magnetic order appears, then we have the right to write down the following expression for the thermal conductivity factor

$$\kappa \approx \frac{\kappa_0}{\left(1-\frac{T}{T_c}+\delta_0\right)^2\sqrt{b\left(1-\frac{T}{T_c}\right)+\frac{H_a}{H_E}}}, \qquad (24)$$

where the coefficient

$$\kappa_0 \approx \frac{\pi\hbar\rho a^{10}\omega_E^2\omega_0^2}{\left(4\gamma S\mu_e^2\right)^2}. \qquad (25)$$

Formulae (24), (25) give an answer to the question posed at the beginning of the article, and the dimensionless parameter δ_0, which appears in the definition M_0 (see formula (4)) according to (24) is $\delta_0 = \frac{H_a}{H_E}$. I. e. it's means that

$$\mathrm{M}_0 = \frac{\mu_e}{a^3}\sqrt{1-\frac{T}{T_c}+\frac{H_a}{H_E}}. \qquad (26)$$

It would seem that when $T = T_c$ we have a small but finable value of the order parameter. However, it should be understood that we are talking about temperatures just below the point of phase transition, that is, it should be considered $T = T_c - \varepsilon$ that, and the field of magnetic anisotropy behaves like a

$$H_a = \begin{cases} 0,\ if\ T \ge T_c \\ H_a\ if\ T < T_c \end{cases}. \qquad (27)$$

That is, near the point of Curie begins to form the energy of magnetic anisotropy and thermal conductivity should increase sharply, which is in accordance with the general principles of the theory of phase transitions. Its maximum value can be estimated as

$$\kappa_{\max} \approx \kappa_0\left(\frac{H_E}{H_a}\right)^{\frac{5}{2}}. \qquad (28)$$

The thermal conductivity of the paramagnetic at $T > T_c$

It is quite clear that in this range of temperatures, not much far from the melting point, the magnetic moment in the absence of an external magnetic field is zero, and therefore no magnetic interactions should not be about, as well as energy of magnetic anisotropy. This means that heat transfer in the paramagnetic phase can only be carried out thanks to acoustic phonons, even if there is no crystalline order. This is clearly a burden of the fact that there is always a heterogeneous density of $\rho(\mathbf{r})$ material, which within the theory of elastic deformation can be presented as a ratio $\rho = \rho_0(1 + div\mathbf{u})$ where $\rho_0 -$ the unperturbed density of the material.

This means that to calculate the only time in this case, the phonon - the phonon relaxation, we can come from an isotropic Hamiltonian species (see. [27])

$$H_{ph} = \frac{\theta}{V}\int_V u_{ik}u_{kl}u_{li}dV \ , \qquad (29)$$

where $\theta -$ some characteristic energy, in order of magnitude corresponding to Debai's energy θ_D. Substituting here (7) with account (8), we find

$$H_{ph} = \sum_{\{\mathbf{q},\alpha\}} \psi(\{\mathbf{q},\alpha\}) b_1^+ b_2 (b_3^+ + b_{-3}) \Delta(\mathbf{q}_1 - \mathbf{q}_2 + \mathbf{q}_3) + c.c. , \qquad (30)$$

where abbreviated designations are introduced $\{\mathbf{q},\alpha\} = (\mathbf{q}_1,\alpha_1;\mathbf{q}_2,\alpha_2;\mathbf{q}_3,\alpha_3)$ и $b_i^+(b_i) \equiv b^+_{\mathbf{q}_i,\alpha_i}(b_{\mathbf{q}_i,\alpha_i})$.

Interaction amplitude is

$$\psi = -i\frac{\theta N}{\sqrt{\omega_1(q_1)\omega_2(q_2)\omega_3(q_3)}}\left(\frac{\hbar}{4\rho V}\right)^{\frac{3}{2}}(e_{1i}q_{1k} + e_{1k}q_{1i})(e_{2k}q_{2l} + e_{2l}q_{2k})(e_{3l}q_{3i} + e_{3i}q_{3l}) . \qquad (31)$$

At the same time, the main Hamiltonian will be

$$H_0 = \frac{1}{N}\sum_{\mathbf{q}} c_s q b_q^+ b_q \ . \qquad (32)$$

According to (30) the integral of the collisions will be

$$L = 2\pi \sum_{\mathbf{q}_2,\mathbf{q}_3,\alpha_2,\alpha_3} |\psi|^2 \{(1+f_1)f_2(1+f_3) - f_1(1+f_2)f_3\}\Delta(\mathbf{q}_1 - \mathbf{q}_2 + \mathbf{q}_3)\delta(\omega_1 - \omega_2 + \omega_3) +$$
$$+2\pi \sum_{\mathbf{q}_2,\mathbf{q}_3,\alpha_2,\alpha_3} |\psi|^2 \{(1+f_1)f_2f_3 - f_1(1+f_2)(1+f_3)\}\Delta(\mathbf{q}_1 - \mathbf{q}_2 - \mathbf{q}_3)\delta(\omega_1 - \omega_2 - \omega_3).$$

And that's why the time we're interested in relaxation will be

$$\frac{1}{\tau_{ph}(q)} = 2\pi \sum_{\mathbf{q}_2,\mathbf{q}_3,\alpha_2,\alpha_3} |\psi|^2 (\overline{f}_3 - \overline{f}_2)\Delta(\mathbf{q}_1 - \mathbf{q}_2 + \mathbf{q}_3)\delta(\omega_1 - \omega_2 + \omega_3) +$$
$$+2\pi \sum_{\mathbf{q}_2,\mathbf{q}_3,\alpha_2,\alpha_3} |\psi|^2 (1 + \overline{f}_2 + \overline{f}_3)\Delta(\mathbf{q}_1 - \mathbf{q}_2 - \mathbf{q}_3)\delta(\omega_1 - \omega_2 - \omega_3). \qquad (33)$$

Because

$$\overline{f}_i \approx \frac{T}{\omega_i(q_i)} ,$$

that of (33) we have

$$\frac{1}{\tau_{ph}(q)} = 2\pi T \sum_{\mathbf{q}_2,\mathbf{q}_3,\alpha_2,\alpha_3} |\psi|^2 \left(\frac{1}{\omega_3} - \frac{1}{\omega_2}\right)\Delta(\mathbf{q} - \mathbf{q}_2 + \mathbf{q}_3)\delta(\omega - \omega_2 + \omega_3) +$$
$$+2\pi \sum_{\mathbf{q}_2,\mathbf{q}_3,\alpha_2,\alpha_3} |\psi|^2 \left(\frac{1}{\omega_2} + \frac{1}{\omega_3}\right)\Delta(\mathbf{q} - \mathbf{q}_2 - \mathbf{q}_3)\delta(\omega - \omega_2 - \omega_3).$$

where $\omega = \omega(q)$.

Taking into account the laws of preservation and after the transition from summation to integration under the rule (18), taking into account the explicit expression for the amplitude of interaction (31) and taking into account the properties of the delta - functions we get as a result of integration

$$\frac{1}{\tau_{ph}(q)} = \frac{2T}{81\pi}\frac{\theta^2 V}{Nc_s^2}\left(\frac{N}{\rho c_s V}\right)^3 q\left(q^3 + q_0^3 - 1{,}5qq_0^2\right) , \qquad (34)$$

where $q_0 = \pi\left(\frac{N}{V}\right)^{\frac{1}{3}}$.

Substituting (34) in the overall expression for the thermal conductivity ratio

$$\kappa_{ph} = \frac{c_s^2}{6\pi^2}\int_0^{q_0} q^2\tau_{ph}(q)dq \ ,$$

will have

$$\kappa_{ph}(T) = \frac{27}{4\pi^2}\frac{\rho^3 c_s^7}{T\theta^2}\left(\frac{V}{N}\right)^{\frac{7}{3}} . \qquad (35)$$

For a numerical estimate κ it can be assume that $\frac{V}{N} \approx \overline{a}^3$, where $\overline{a} -$ is some average interatomic distance. Substituting in (35) specific numerical values

$$\rho \sim 10\frac{\tilde{a}}{\tilde{n}\grave{i}^{\ 3}} ,\ \tilde{a}_{\grave{i}} \sim 10^5\frac{\tilde{n}\grave{i}}{\tilde{n}} , K\overline{a} \sim 10^{-8} \acute{y}\eth\tilde{a} \quad \sim 10^3\ K \sim 10^{-13}\ \acute{y}\eth\tilde{a}\ \ \theta \sim 10^2 \quad \sim 10^{-14} \quad ,$$

we get from here $\kappa_{ph} \approx 10^{23}\frac{1}{\tilde{n}\grave{i}\ \cdot \tilde{n}}$. If put in (35) $T = T_c$ and compare with (28), we'll see that

$$\kappa_{\max} > \kappa_{ph}(T_c) . \qquad (36)$$

Therefore, the jump in thermal conductivity in the transition from the paramagnetic phase to the magnetic will be the magnitude of

$$\Delta = \kappa_{\max} - \kappa_{ph}(T_c) . \qquad (37)$$

Ratios (36), (37) allow us to build dependence $\kappa(T)$ in a wide interval of temperature up to melting temperature, which is reflected in (Figure 1).

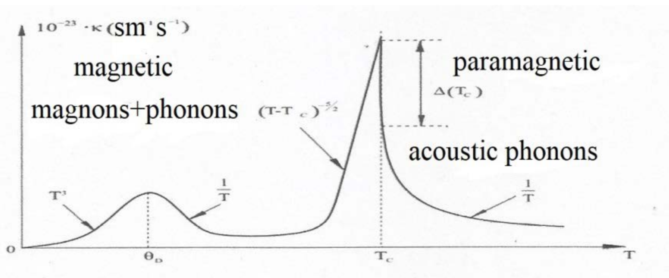

Figure 1: Schematic illustration of temperature dependence of ferromagnets in wide diapason of temperature, including the Curie's point.

Conclusion

In conclusion, it is worth paying attention to some of the results above.

1. Using a quasi-classical kinetic equation, the thermal conductivity ratio of ferromagnetic in the vicinity of phase temperature is calculated;

A critical staid growth index has been found and it's shown that

$\kappa = \kappa_0 \left(1 - \frac{T}{T_c} + \frac{H_a}{H_E}\right)^{-\lambda}$, where $\lambda = \frac{5}{2}$ (see expr. (24)).

A comparison of the phonon thermal conductivity in the paramagnetic phase $T > T_c$ with thermal conductivity in the magnetic phase at $T \leq T_c$ and calculated the jump in thermal conductivity at the point $T = T_c$.

A graphic interpretation of the result in a wide range of temperatures is given.

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