International Journal of Advance Innovations Thoughts & Ideas

Abood, Int J Adv Innovat Thoughts Ideas 2022, 11:3

Research Article Open Access

Study of Pion-Nucleus Scattering Lengths within Alpha-Cluster Model

Saad N Abood*

Departments of Physics, University of AL-Nahrain, Baghdad, Iraq

***Corresponding author:** Saad N Abood, Departments of Physics, University of AL-Nahrain, Baghdad, Iraq; E-mail: ner_ner2@yahoo.com

Received: 08-Feb-2022, Manuscript No. IJAITI-22-52742; **Editor assigned:** 11-Feb-2022, PreQC No. IJAITI-22-52742 (PQ); **Reviewed:** 25-Feb-2022, QC No. IJAITI-22-52742; **Revised**: 08-Apr-2022, Manuscript No. IJAITI-22-52742 (R); **Published:** 13-Apr-2022, DOI: 10.4172/2277-1891.1000176

Citation: Abood SN (2022) Study of Pion-Nucleus Scattering Lengths within Alpha-Cluster Model. Int J Adv Innovat Thoughts Ideas 11: 176.

Abstract

In terms of pion-alpha scattering lengths, pion-nucleus s-wave scattering lengths are estimated from a multiple scattering series. For all nuclei with there is a level of agreement with the values derived from mesic-atom data of 15%-25%. This supports the theory that pion absorption on light nuclei is predominantly due to alpha-particle substructures.

Keywords: Pion-alpha scattering lengths; Cluster model

Introduction

Particle absorption in nuclei is still largely a mystery. A free nucleon cannot absorb a pion because of energy-momentum conservation. Although the one-nucleon mode is not impossible when the nucleon is bound, it appears to be inhibited in comparison to reaction mechanisms involving at least two nucleons. Attempts to calculate the two nucleon processes, however, have only been partially successful. Calculated absorption rates for light nuclei are two to three times lower than those derived from experimental widths of pionic atoms. It's also difficult to understand the decay products of pionic atoms in terms of basic π NN→NN amplitude without significant final state interactions, because deuterons, tritons, and other multi nucleon events are observed in addition to the predicted two nucleon mode [1]. The notion of pion absorption by an alpha particle cluster in the nucleus has been proposed several times to explain these unusual results, but further calculations have only produced semi-quantitative agreement [2]. The investigation of nuclear gamma rays from pion absorption at rest and in flight has lately provided evidence that alpha clusters are important: strong lines corresponding to excited states of the nucleus minus an alpha particle appear to always be observed in doubly even nuclei. As a result, considerable portions of pion capture events by ^{16}O result in a 4.4 MeV level of ^{12}C. The goal of this research is to take the alpha cluster model seriously and match the energy shifts and widths of pionic atoms' 1 s levels to those seen in the alpha particle [3]. For most stable isotopes, the widths and shifts of the 1 s levels have been measured with, and the usual explanation is in terms of a complex velocity dependent optical potential [4].

For most stable isotopes with a ≤ 20, the widths and shifts of the 1 s levels have been measured, and the usual explanation is in terms of a complex velocity dependent optical potential. This takes the form of an isoscalar nucleus:

$$2\mu V = -4\pi\left\{(1+\mu/m)b_0\rho(r)+(1+\mu/2m)B_0\rho^2(r)+\nabla q(r)/\left(1+\frac{4}{3}\pi q(r)\right)\nabla\right\} \quad \ldots\ldots (1)$$

The nuclear density is ρ(r), while the pion and nucleon masses are μ and m, respectively. The density of the gradient potential includes linear and quadratic elements [5].

$$q(r) = c_0\rho(r)/(1+\mu/m)+C_0\rho^2(r)/(1+\mu/2m) \quad \ldots\ldots\ldots (2)$$

The true parameters b_0 and c_0 should be near to the s and p wave isoscalar pion nucleon scattering lengths, with binding and correlation effects possibly modifying them, but there is little understanding of the physical relevance of the complex parameters B_0 and C_0. They are used to account for the two nucleon absorption, among other things. Another disadvantage is that fitting this potential to an ensemble of unique nuclei yields parameters that are not actually constant, implying that the form of the potential in Equation (1) is not entirely right. The issue is most likely with the quadratic terms in the density, which is especially concerning because, because b_0 is tiny, the quadratic term plays a significant role in the local section of the potential [6].

The relationship between the length of pion-nucleus scattering and energy shifts

There is a well-known relationship between the complex energy shift and the s-wave scattering length a_s (assessed in the center of mass frame) in the limit when the range of the pion nucleus strong interaction potential is short:

$$E_{1s} - \frac{1}{2}i\Gamma_{1s} = -(4\pi/2\mu_R)a_s|\Psi_{1s}(0)|^2 \quad \ldots\ldots\ldots (3)$$

The decrease pion mass is denoted by μR. A pion in a 1 s Coulomb orbit can be thought of as suffering repeated s-wave scattering from the nucleus in this procedure. The pion hydrogenic wave function is Ψ_{1s}, and $|\Psi_{1s}(0)|^2$ is the chance of locating the pion in the nucleus region [7].

This suggests that, even in the more general scenario of finite-size nuclei, the complex energy shift in the s-wave orbit determines virtually entirely the s-wave scattering lengths. We want to use the alpha particle scattering lengths *a(πα) a(πα)* as an input in a multiple scattering calculation to find the scattering lengths in other nuclei because the pion nucleus scattering lengths *a(πA)* can be found this way. To do so, we must extract the scattering lengths with greater precision than is achievable using Equation (3). There is no model-independent approach for removing the Coulomb potential's effects. The assumed form of the pion nucleus potential will influence the relationship between E_{1S}, Γ_{1S} and a_s to some extent [8]. We believe that the generic form of the optical potential in Equation (1) is theoretically valid. As a result, we fitted the parameters to precisely reproduce the observed s-wave shifts and widths for each nucleus. Fits to p-wave data from heavier nuclei were used to calculate the gradient potential's parameters [9]. The s-wave scattering lengths a_s might then be estimated using the same parameters by turning off the Coulomb potential, providing the $a_s^{(1)}$ of Table 1. The gradient was set to zero, the parameters of the local potential were changed to give the right complex 1 s shifts again, and the s-wave scattering lengths were $a_s^{(2)}$ recomputed to check the model dependence of this solution [10]. Because the non-local potential has a minor influence on E_{1S}, and Γ_{1S}, the difference $a_s^{(2)}$ -$a_s^{(1)}$ is not particularly great. As a result, the difference does not reflect the complete uncertainty in this model's scattering length extraction. In Table 1, we also provide the findings of a version of Equation (1) that takes into account the finite size effect:

$$E_{1s} - \frac{1}{2} i\Gamma_{1s} = -(4\pi / 2\mu_R) a_s^{(3)} \int \rho(r) |\Psi_{1s}(r)|^2 d^3 r \quad \ldots\ldots\ldots\ldots (4)$$

The shift is defined with regard to the appropriate energy eigenvalues is the Coulomb wave function corresponding to a spread-out charge density. The differences between the more precise determinations and the derivation are fairly modest [11].

Results and Discussion

The alpha cluster model for light nuclei is now presented in its most basic form, as described by Inopin. Even nuclei are thought to be made up of alpha particles that are solely in their ground states. Except in the case of a short-range repulsion between two alpha particles, anti-symmetrization between nucleons in different clusters is ignored. Furthermore, the alpha particles will be assumed to be fixed at the vertices of regular geometric forms [12]. For ^{12}C, for example, three alpha-particles are placed at the vertices of an isoscalars triangle with a side length of 1.3 fm. It was proven that this reproduced not only the elastic form factor of ^{12}C, but also the form factor of the first excited state (J^P=2$^+$at 4.4 MeV) fairly effectively. As a result of pion capture in ^{16}O, the emergence of such collective states is quite normal in this type of model. Excitation of single particle states, which corresponds to the excitation of one of the alpha particles, would be far more difficult to explain. A second observation is that there is only a small overlap among the clusters due to the great distance between their centers. When we consider scattering from two fixed centers that do not overlap, the scattering amplitude for the composite system is solely a function of the physical individual amplitudes for the same incident energy [13]. The s-wave scattering lengths of such a system, in particular, are solely determined by the constituent's s-wave scattering lengths. We don't need to know anything about the pion-alpha interaction's behavior or the p-wave amplitudes. It is simple to write down a multiple scattering series for the π^{12}C scattering length while working inside such a model [14].

$$a_s(\pi^{12}C) = 3a_s(\pi\alpha) + 3\times 2a_s^2(\pi\alpha)/d + 3\times 2\times 2a_s^3(\pi\alpha)/d^2 + \ldots \quad \ldots\ldots (5)$$

The underlying assumption in this type of scattering theory is that the pion velocity is greater than that of alpha-particles in the nucleus. As a result, contributions where the ^{12}C nucleus is excited into a rotating state (e.g., 2$^+$ at 4.4 MeV) and then de-excited by a second contact are included in the double scattering. If we assume the opposite limit, where the rotating velocity of the ^{12}C triangle is very big, the 1/d would be substituted by the average value of the inverse spacing of two points on a sphere of radius R. This equates to simply preserving the ^{12}C ground state between scatterings [15]. The twofold scattering in the latter scenario would be three times greater than in the former. The amplitudes must be evaluated in some common rest frame, such as the laboratory, to account for the effects of recoil in Equation (5) In Table (1) the cm. amplitudes should be multiplied by (1+μ/M) to obtain the laboratory amplitudes before applying Equation (5) [16].

The series converges extremely quickly since $a_s(\alpha\pi)$ is so small (1/7 fm in magnitude). The second term is damaging and has a magnitude of roughly 10%, but the third has a magnitude of just about 1%. We can imagine ^{16}O as a normal pyramid, and ^{20}Ne as a regular bi-pyramid of alpha particles with side lengths of d=3.6 fm and 3.3 fm, respectively, to obtain the right nuclear radii. The trivial combinational factors are the main change in the multiple scattering series [17]. Table 1 shows the scattering lengths estimated this manner. In most cases, the agreement with the experimental values is less than 10%. It tends to lie within the error bars for the absorptive part, whereas the computed value for the real part has a systematic tendency to be too small in magnitude. This could be due to a flaw in the theory or the data. It's worth noting that for the real components, a method that ignores multiple scatterings entirely would be even better.

We studied the remaining experimental data in terms of alternative simple cluster models, encouraged by our success with the 4 n nuclei. For example, ^{9}Be was assumed to be a pair of alphas separated by 3.6 fm with a neutron stuck in the center. The evaluation employed a neutron scattering length of a_n=-0.092 ± 0.002 μ^{-1}

An alpha-deuteron combination separated by a distance of d=3.6 fm is a commonly used model for ^{6}Li. We used the ^{6}Li scattering length to compute the scattering lengths of deuterium and the other doubly odd nuclei because the true part of the deuteron scattering length has yet to be precisely determined. Within a very significant error, the real part of our deuteron value coincides with the experimental one [18]. However, there is a clear standard deviation difference for the imaginary part. The experimental value of this latter has been calculated from the cross section for P+P → π^++*d* , allowing for the potential of radiative capture [19]. The geometric structure of the ^{9}B and 14N was supposed to be the same as that of the ^{12}C and ^{16}O, with one of the alphas replaced by a deuteron. The imaginary parts of the scattering lengths are well reproduced, but the real parts have a noticeable inaccuracy of 20%. The ^{11}B and ^{19}F results were calculated by extracting a triton scattering length from the ^{7}Li in the same way as the deuteron was taken from the ^{6}Li [20-25].

Nucleus	ϵ_{1s} (keV)	Γ_{1s} (keV)	$a^{(1)}$ (Exp.)	$a^{(2)}$ (Exp.)	$a^{(3)}$ (Exp.)	a (Cal.)	Discrepancy $a^{(1)}$ -a
^{4}He	-0.075 (20)	0.045 (3)	-98+i30	-98+i30	-105+i33		- (42 ± 11) + i(16 ± 7)
^{12}C	-5.874 (92)	3.14 (12)	-320+i94	-330+i94	-322+ i104	-278 + i78	- (24 ± 14) + i(9 ± 11)
^{16}O	-15.033 (240)	7.64 (49)	-378+i109	-401+i109	-384+ i100	-363 + i100	- (44 ± 23) + i (5 ± 27)
^{20}Ne	-33.34 (50)	14.5 (3)	-482+i121	-518+i128	-475+i126	-438 + i116	- (11 ± 6) + i (6 ± 4)
^{9}Be	-1.595 (9)	0.59 (14)	-279+i54	-283+i54	-281+i59	-268 + i48	
^{6}Li	-0.324 (3)	0.195 (12)	-130+i39	-130+i39	-137+i42		
^{2}H			-49+i(4 ± 0.4)			-31+i9	- 18 - i(5 ± 4)
^{10}B	-2.977 (85)	1.59 (11)	-273+i78	-278+i78	-275+i85	-220+i63	- (53 ± 9)+i(15 ± 6)
^{14}N	-9.915 (144)	4.34 (24)	-357+i88	-370+i87	-358+i94	-302+i83	- (53 ± 10)+i(5 ± 10)
^{3}Li	-0.57 (4)	0.195 (13)	-228+i40	-230+i49	-233+i44		
^{3}H						-135 ± 4+i(14 ± 4)	
^{11}B	-3.839 (85)	1.79 (12)	-355+i90	-363+i90	-353+i102	-310+i62	- (45 ± 9)+i(28 ± 7)
^{19}F	-24.46 (35)	9.4 (1.5)	-477+i100	-502+i113	-464+i103	-470+ i102	- (7 ± 17)-i(2 ± 18)

Table 1: Shows the scattering lengths.

Conclusion

Table 1 show that the overall agreement is better than 20%, probably better for the alpha particle nuclei than the other. There isn't a single free parameter that can be tweaked. What are the implications of this? For all nuclei, the first term in Equation (5) multiple's scattering series dominates the result. It is not reliant on any input from nuclear structures. The nuclear structure model only appears in the rescattering term, i.e. at a 10% level. As a result, the results are relatively unaffected by the geometric model used for the alpha structure. Furthermore, the rescattering terms do not necessarily increase theory-experiment agreement. This could be attributable to systematic errors in the experimental values of $a_s(\pi A)$, as well as the presence of other factors of order $a_s(\pi\alpha)^2$. Despite the fact that our naive alpha-cluster model cannot quantitatively quantify the deviations from the first order term in Equation (4), it accurately predicts that the deviation will be modest.

The minimal rescattering is due to the strong repulsive alpha-alpha interaction, which keeps these particles roughly 3 fm apart in light nuclei. A quasi-deuteron model, as an alternative to the alpha cluster model, reproduces the single scattering term of our expansion by definition, but it presents no reason why the first should be a decent approximation. In addition, the imaginary part of this "deuteron" dispersion length is 3-4 times that of the real deuteron of course, delicate cancellations are observed in the calculations of pion absorption by real deuterons. If pions interact with light nuclei mostly through alpha cluster members, as we explain in this paper, we should expect a comparable distribution of fast recoil particles from ^{4}He and ^{12}C, ignoring final state interactions. The reaction on ^{4}He should be investigated further both theoretically and empirically, as it appears to be a typical light nucleus in terms of pion absorption. Finally, the analysis calls into question Equation (1) assumption's that the imaginary part of the optical potential should be proportional to the square of the nuclear density. It's possible that a single linear in cluster density would be more suited.

References

1. Huber MG (1970) Short range correlations in Nuclei. Ann de Phys 14: 239-253.
2. Dover CB, Lemmer RH (1971) Pions in nuclear matter-an approach to the pion-nucleus optical potential. Ann Phys 66: 248-292.
3. Krell M, Ericson TEO (1969) Energy levels and wave functions of pionic atoms. Nuclear Phys B 11: 521-550.
4. Tauscher L, Schneider W (1974) Optical potential calculations for the 1s level in pionic atoms. Zeitschr Fur Phys 271: 409-415.
5. Ziock KOH, Cernigoi C, Gorini G (1970) Pion capture in 4He. Phys Lett B 33: 471-473.
6. Bellotti E, Cavalli D, Matteuzzi C (1973) Positive-pion absorption by C nuclei at 130 MeV. Nuovo Cimento A 18: 75-93.
7. Lewis CW, Ullrich H, Engelhardt HD, Boschitz ET (1973) Possible alpha-particle cluster absorption of pions by 16O. Phys Lett B 47: 339-341.
8. Lind VG, Plendl HS, Funsten HO, Kossler WJ, Lieb BJ, et al. (1974) Multinucleon Removal in 220-MeV π-Reactions with A<~ 51 Nuclei. Phys Rev Lett 32: 479.
9. Ashery D, Schiffer JP (1986) Pion absorption in nuclei. Annu Rev Nucl Part Sci 36: 207-252.
10. Shapiro IS, Kolybasov VM, Augst GR (1965) Treiman-Yang criterion for direct nuclear reactions. Nucl Phys 61: 353-367.
11. Hüfner J (1975) Pions interact with nuclei. Phys Rep 21: 1-79.
12. Kossler WJ, Funsten HO, MacDonald BA, Lankford WF (1971) Nuclear deexcitation γ Rays in N 14, C 14, and N 15 following π-capture on O 16. Phys Rev C 4: 1551.
13. Deser S, Goldberger ML, Baumann K, Thirring W (1954) Energy level displacements in pi-mesonic atoms. Phys Rev 96: 774.
14. Seki R, Cromer AH (1967) Analysis of π-mesonic atoms. Phys Rev 156: 93.
15. Sauer PU (1974) Can the charge symmetry of nuclear forces be confirmed by nucleon-nucleon scattering experiments?. Phys Rev Lett 32: 626.
16. Hüfner J, Tauscher L, Wilkin C (1974) An analysis of pion-nucleus scattering lengths within an α-cluster model. Nucl Phys A 231: 455-461.

17. Backenstoss G, Egger J, Von Egidy T, Hagelberg R, Herrlander CJ, et al. (1974) Pionic and muonic X-ray transitions in liquid helium. Nucl Phys A 232: 519-532.
18. Backenstoss G (1970) Pionic atoms. Annu Rev Nucl Sci 20: 467-508.
19. Eckhause M, Kane FR, Kane JR, Martin P, Miller GH, et al. (1972) Strong interaction effects in the 2p-1s transition in pionic neon. Nucl Phys B 44: 83-92.
20. Ikeno N, Molina R, Oset E (2021) Triangle singularity mechanism for the p p→ π+ d fusion reaction. Phys Rev C 104: 014614.
21. Inopin EV, Tishchenko BI, Shebeko AV (1966) Description of inelastic diffraction scattering by the complex angular momentum method. Soviet Phys JETP 22: 1247.
22. Beg MAB, Furlong RC (1985) λ φ 4 theory in the nonrelativistic limit. Phys Rev D 31: 1370.
23. Babenko VA, Petrov NM (2019) On the breakdown of charge independence and charge symmetry of the pion-nucleon coupling constant. Phys Atom Nuclei 82: 952-962.
24. Ryan JW (1963) Branching ratios of reactions of π-mesons stopped in hydrogen and deuterium. Phys Rev 130: 1554.
25. Niskanen JA (1996) Comment on ‘‘Role of heavy meson exchange in near threshold NN→dπ’’. Phys Rev C 53: 526.