

Bearing Capacity Evaluation of Footing on a Layered - Soil using ABAQUS Mosadegh A* and Nikraz H

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Abstract

In this paper, finite element method (FEM) is applied to calculate bearing capacity of a strip footing on one-layer and two-layer soil. To investigate the effect of various parameters on soil failure mechanism under the footing a commercial finite element software, ABAQUS, has been used. Soil profile contains two soil types including sand and clay. Soil behaviour is represented by the elasto-plastic Drucker-Prager model and footing material is assumed isotropic and linear elastic. For a homogenous soil profile, the effect of soil properties such as dilation angel, initial condition and footing roughness on soil failure mechanism under the footingare assessed. For a one-layer case, the bearing capacity also is calculated which has a good agreement with Terzaghi's equation. For a layered soil, soft-overstrong soil, the effect of layer thickness, soil shear strength and material property on bearing capacity value and failure mechanism of footing is investigated. It is concluded that the bearing capacity of footing decreases as the height of clayey soil increases whilst the displacement under footing increases. There is a critical depth where the stronger bottom layer does not affect ultimate bearing capacity and failure mechanism of footing.

Keywords: Bearing capacity; FEM; strip footing; one-layer soil; two-layer soil

Introduction

Geotechnical engineers often should solve problems in layered soil while the majority of existing studies have mostly focused on homogeneous continuum [1,2]. Predicting ultimate bearing capacity of footings on layered soil is very important as it is a requirement for any design and the failure mechanism of soil under footing and the bearing capacity value mainly depend on soil properties of each layer and the layer thickness. Terzaghi and Peck for the first time in 1948 [1] analysed strip footing behaviour on sand overlaying clay which followed by many researchers. Methods of solving bearing capacity of footing are classified in four major approaches: limit equilibrium, limit analysis approach, semi empirical approach and finite element method. In recent years, Finite Element Method (FEM) have been widely using in geotechnical studies to investigate soil behaviour[3-6]. In practice, for bearing capacity analysis engineers are seeking less complicated solutions to simplify computations as experimental analysis is time consuming and commonly used solutions such as limit equilibrium are no longer applicable. Therefore, computer programs developed based on the finite element method have been receiving much attention over recent decades as the powerful tool for solving complex cases. Hence, the application of FEM to evaluate bearing capacity evaluation of a footing on a layered soil is the objective of this study.

In the current study after describing problem definition, the bearing capacity analysis of a strip footing on one-layer soil will be presented. In addition, the effect of different parameters on soil failure mechanism and on bearing capacity value will be discussed. Then, in the following section the bearing capacity analysis of a two-layer-soil will be presented and the results will be compared with the results from the literature. Computations will be carried out using a commercial finite element software ABAQUS, version 6.13[7]. Finally, the conclusions and final remarks will be discussed in the last section of this study.

Materials and Methods

Following section presentes problem definition and the methodology used for modelling of footing. Due to the long length of the foundation compared to its width, the problem can be analysed assuming plane strain conditions and because of symmetry only half

of the system can be modelled.

Two different main cases will be investigated in this paper. In the first case, it is assumed that foundation is resting on one-layer soil and in the second one there is a two-layer system. For the first case, it is assumed that the one- layer system is sandy soil and footing overlays on Soil (2). For this case, to validate FE results, bearing capacity value of soil will be compared with Terzaghi calculation [1]. In addition, the effect of three different parameters on soil behaviour under the footing will be considered for one-layer system. The parameters those effects will be taken into considerations are footing type, dilation angel and initial condition. For analysing the effect of footing, once the footing will be modelled as a rigid rough footing with no horizontal movement of soil immediately under footing and once as a rigid smooth one which implies horizontal soil movements at soilfooting interface. Then, for invesigating the effect of dilation angel the effect of three dilation angel of 0°, 10 ° and 25° will be taken into considerations.



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And in the third case the effect of K_0 or initial conditions on bearing capacity of soil will be studied considering K_0 values of 0.4, 1 and 10.

For analysing two-layer soil as a second case it is assumed that the clayey soil is resting on top of the sandy soil and the footing is sitting on top of clayey soil. Based on Figure 1, Soil (1) is clayey and Soil (2) is sandy. It should be pointed out that for this case the effect of clay depth on bearing capacity value will be investigated considering clay depth, h₁, has different thicknesses and h₁/B has the value of 0.15, 0.5, 1 and 1.5 in which B is total width of footing. For one–layer case soil is modelled as an isotropic ealsto-plastic material satisfying Drucker-Prager failure criterionadapted from Helwany which presented in Table.1[8]. For comparing with Terzaghi's equation for a one-layer sand, it is assumed that soil layer is replaced with the overburden of $q = \Upsilon$.D=9.60 kPa due to a 0.5-m-thick foundation. For the top clayey layer soil parameters are adapted from Ziaei et al. [9].

It is worth noting that for plane strain considerations, the Mohr-Coulomb parameters and Drucker-Prager parameters are converted to each other based on existing formulations as follows [8]:

$$\tan\beta' = \frac{3\sqrt{3}\tan\phi'}{\sqrt{9+12}\tan^2\phi'} \text{ and } d' = \frac{3\sqrt{3}C'}{\sqrt{9+12}\tan^2\phi'}.$$

 β' and d' are representing friction and cohession in Drucker-Prager model. As is illustrates in Figure 2, only half of the system is modelled due to symmetry. The length and height of the model are large enough to keep the boundary conditions away from affecting soil behaviour incorrectly. It should be noted that in this study, the X-Y plane is the area in which the soil is subjected to various loads, e.g. positive direction for Y is opposite direction of the weight. For boundary conditions, as shown in Figure 2, vertical side of the model is fixed in a horizontal direction with vertical displacement, and the bottom of the model is fixed in both vertical and horizontal directions. In all models, the mesh has been refined in areas with stress concentration under and near the footing, which leads to more accurate answers. However, the smaller the elements are, the larger the computational time is needed. For this purpose and to achieve the reasonable number of meshes, mesh convergence study was carried out. It was found from the results that a model of 510 elements is accurate enough for this problem. The element adjacent to the footing, as shown in Figure 2, has the width of w=0.13m which is small enough for analysis according to Day and Potts [10].

Table 1: Material Pro	operties of Sandy	soil-Drucker-Prage
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Type of soil	Term	Value
Soil (2)	Density, Y (kg/m ³)	1920
	Young's Modulus, E' (MPa)	182
	Poisson's Ratio, 9	0.3
	Cohesive strength, d' (kPa)	<1
	Friction angle, β' (plane strain), (deg)	46°
	Dilation angle, ψ (deg)	4°
	Flow stress ratio, K	1

Table 2: Material Properties of clayey soil Mohr-Coulomb

Type of soil	Term	Value
Soil (1)	Density, Υ (kg/m ³)	1600
	Young's Modulus, E' (MPa)	5
	Poisson's Ratio, 9	0.3
	Cohesive strength, C' (kPa)	20
	Dilation angle, ψ' (deg)	5°
	Dilation angle, ψ,(deg)	1°

The model is created in three steps. In the first step which is the initial condition, all boundary conditions are defined as described previously and surcharge load is applied on top of the model. In the next step, a geostatic step is applied in which the gravity load is applied to the model. In the third step, a downward movement of δ / B=0.1 is applied on top of the soil under footing where δ is vertical displacement and B is the width of footing. It should be noted that the duration for this lstep is 100 seconds to avoid sudden collapse of soil body. Moreover, it is assumed that relative movement between soil and footing is impossible.

During the generation of initial condition and stress prior to loading of footing, a lateral pressure coefficient of K_0 is calculated based on following formulation for sand and clay. According to Jaky's formula $K_0=1-\sin\Phi$ for the sand and $K_0=0.95-\sin\Phi$ for the clay [11]. So for this study K_0 is 0.4 for the sand and 0.86 for the clay due to 37.5° and 5° friction angel of sand and clay, respectively. A short-term stability of footing in particular is considered, so the sand is assumed to be fully drained and the clay is considered to be undrained.

Results

In the following sections the effect of different parameters summarised in Table 3 are investigated on the failure mechanism and bearing capacity of footing. Firstly, the bearing capacity assessment of one-layer soil will be presented and the effect of parameters variation on one-layer soil behaviour will be discussed and then in the following section for a two-layer soil, the effect of soil parameters and depth of top layer on bearing capacity value and on failure mechanism will be argued.

Table3. Variables of the study				
Different footing configurations	Constant parameter	Variable parameter		
Case I One-Layer: sand	Geometry of model, Width of footing Soil properties-Table1	Footing roughness and properties Dilation angel ψ =0°; 10°; 25° Initial condition; K ₀ =0.4; 1;10		
Case II Two-Layer: Clay-over-Sand	Width of footing	$\frac{h1}{n} = 0.15; 0.5; 1; 1.5; 2$		
	Soil properties-Tables 1 and 2	^B C _u		

 $\frac{h_1}{B}$ =The ratio of clay depth to footing width

Bearing capacity ealuation of of footing on one-layer soil

As discussed earlier, the footing undergoes a downward movement of $\delta/B=0.1$ during 100 seconds while in the beginning of the analysis there is only gravity load and surcharge applying to the soil body. This downward movement leads to an increase in pressure under the footing up to failure point. In Figure 3-a general shear failure of soil under footing based on Terzaghi model is illustrated. It can be easily noticed from Figure 3-a that there are three different distinct area zones under the footing at failure point: triangular zone immediately under the footing; two radial zones, and two Rankine passive zones[1]. The result of plastic shear at failure point of footing in the present study is illustrated in Figure 3.b. Immediately the existence of different areas in failure zone can be noticed under the footing which is in a good agreement with the failure mechanism suggested by Terzaghi. Terzaghi also derived the bearing capacity equation for a shallow strip footing on a thick layer of homogeneous soil based on general shear failure:

 $\begin{aligned} q_u &= C' N_c + q N_q + 0.5 \gamma B N_{\gamma} \\ Nq &= e^{\pi \tan \varphi} tan^2 (45 + \frac{\varphi'}{2}) \\ Nc &= (Nq - 1) Cot \, \Phi' \\ N\gamma &= (Nq - 1) tan \, 1.4 \, \varphi' \end{aligned}$

C' is cohesion, q is overburden pressure, Nc, Nq and N are non-dimensionl bearing capacity of footing as a function of φ .









Figure 4 illustrates pressure–settlement curve results through FEM analysis and the results are compared with bearing capacity calculated based on Terzaghi method. It should be noted that for FEM analysis the curves are based on the results using Drucker-Prager model, once dilation angel is taken into consideration and once without considering that. It can be seen that in FEM analysis with considering dilation angel the bearing capacity value is 2200 kPa while in the case of no dilation angel the predicted bearing capacity is 1900 kPa. In other words, the bearing capacity in FEM analysis with dilation angel is 13% higher than those with no dilation angel. The bearing capacity calculated through Terzaghi equation is 2124 kPa which is slightly smaller than those obtained by FEM analysis with dilation angel and is

higher than those calculated by FEM analysis without dilation angel. The main reason of this difference can be due to different assumptions in the methods used, e.g. in Terzaghi's equation soil is assumed to be perfectly plastic while in current finite element analysis soil is an elasto-plastic material. The results found in the current study are in good agreement with the results achieved by other researchers in the past[12] [13]. It should be noted in this section the dilation angel is assumed to be 4° which is equal to ϕ -34° [14].

Effect of footing roughness on failure mechanism and soil settlement

To evaluate the effect of soil interface on soil failure mechanism, three analyses are carried out for the case of non-dilatants one-layer soil. The results for smooth and rough interface of a rigid and flexible footing are presented in Figure 5. In all cases footing is subjected to a load control situation (Δ Fy) and Δ U and Δ v are representing horizontal and vertical movement of footing. As can be seen in Figure 5 horizontal movement occur at the soil interface immediately under the smooth footing, Figures 5-a and 5-c, while for the rough footing there is no horizontal movements due to boundary conditions, Figure 5-b. In addition, for the flexible footing maximum settlement occurs at the edge of footing, 5-c, while for rigid footing it happens under footing considering longer arrows show maximum displacement. It is clear that soil failure mechanism in rigid case for rough footing, Figure 5-b, is deeper and wider than those for smooth ones. These results are in a good agreement with the results presented by other researchers in the past[15]. It should be noted that the effect of interaction between two parts was not taken into consideration.

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Effect of dilation angel on failure mechanism

The vectors of incremental displacement for the last increment of a smooth footing are shown in the Figure 6. As can be seen with increasing dilation angel from 0° to 25° the failure mechanism width increases from 2.5 B to 8B. In other words, for all cases the failure mechanism is deeper and wider when dilation angel is higher. In addition, the higher the dilation angel is, the more the tangential angel appears besides footing edge. As was illustrated in Figure 4 dilation angel has an important effect on the bearing capacity value showing a satisfactory agreement with results from the literature [16].

Effect of initial condition on failure mechanism

To investigate the effect of K_0 or initial condition, the analysis with rigid footing is repeated with K_0 value of 1.0, and 10 for smooth footing with no dilation angel. The results of load- displacement curves are shown in Figure 7. It is evident that the value of K_0 has an impact on load displacement curve prior to failure but not on ultimate bearing capacity value.

Evaluation of bearing capacity in a two-Layer soil

In the following section, ultimate bearing capacity prediction of a strip footing on two-layer soil is presented. Footing material is assumed to be linear-elastic, rigid, rough resting on a two-layer system in which top layer is clayey soil with soil parameters matches to Mohr-Coulomb plasticity presented in Table 2. It should be noted that the parameters of a Drucker-Prager and Mohr-Coulomb can be converted to each other based on existing formulation presented earlier. Soil parameters for the bottom layer or sand are based on Table 1. However, those parameters were converted to Mohr Coloumb and parameters for sand are chossien of zero and friction of 37.5°. Other elastic parameters are based on Tables 1 and 2.

Effect of clay depth on bearing capacity and displacement

This section investigates the effect of clay thickness at five different clay depth of h_1 /B varying between 0.15, 0.5, 1, 1.5 and 2 times footing width on bearing capacity value and on vertical settlement of footing and results are shown in Figure 8.

Figure 8.a illustrates the effect of adding clay on bearing capacity reduction of the footing. As is shown bearing capacity drops from 1900 kPa to 530 kPa by adding 45 cm of clay layer, $h_1/B = 0.15$, showing a dramatic fall of %70 in bearing capacity reduction. Figure 8.b shows the effect of change in clay depth on bearing capacity by considering $h_1/B = 0.15$, 0.5, 1, 1.5 and 2. It can be seen that the bearing capacity drops from 530 kPa to 250 kPa when h_1/B increases from 0.15 to 0.5. Increasing h_1/B from 0.5 to 1.5 leads a reduction of 100 kPa in bearing capacity and bearing capacity goes down to 150 kPa at $h_1/B=1.5$. After this point increasing h_1/B has no effect on bearing capacity. In other words, the depth for which the bottom layer strength does not affect the bearing capacity value of the entire model is when h_1 reaches 4.5 m or $h_1/B=1.5$. Figure 8.c presents the effect of

clay thickness on soil displacement under the footing. For this purpose, 140 kPa of pressure has been applied on top of footing to analyse soil displacement behaviour under the same load pressure at different clay depths. This pressure has been selected due to weight of footing plus weight of any machinery on top of it. With increasing depth of clay from $h_1/B=0.15$ to 1.5, the vertical settlement under footing increases from 8 to 20 cm and increasing h_1/B from 1.5 to 2 causes a reduction in displacement from 20 to 18 cm showing displacement does not increase after $h_1/B=1.5$. The displacement reaches its peak at depth of 4.5 m or when $h_1/B=1.5$. These results have a good agreement with results published by other researchers [1], [14], [9]. So far the effect of various parameters on bearing capacity of footing on a layered soil has been investigated. The effect of both initial condition and depth of clayey soil on bearing capacity of





h1/B

0.5

0.15 0

02

1.5

layered soil has been compared and results are shown in Figure 9. It can be seen that when $h_1/B= 0.15$ or is constant, bearing capacity varies between 1800 to 1920 kPa considering K0 varies between 0.4 to 10. When K0 is constant at 0.4, the bearing capacity decreases dramatically from 1900 kPa to 150 kPa when h₁/B increases from 0 to 2. It means that depth of clay layer has much more effective influence on bearing capacity of footing compared with initial condition.

Effect of soil layer parameters on vertical stress distribution

Figure 10 provides increase in vertical stress as a function of depth directly under the footing from z=0 to 8m which z is representing the depth. Increase in vertical stress distribution calculation is carried out for two cases based on FEM. For FEM analysis, the first case is a homogenous case considering $E_2/E_1=1$ and the results will be compared with those calculated from Boussinesq solutions [18]. And the second one is a two-layer soil when top layer is clayey soil with $E_2/$ E1=30, considering h1=0.45. To validate the results with Boussinesq solution in FEM analysis it is assumed soil is weightless. At top of the soil profile the load of q=12 KN/m² or 12 kPa is applied which is due to weight of a 3-m-wide concrete footing. The increase in vertical stress under centre of footing is plotted and results are shown in Figure 9. It can be seen that for a one-layer soil both diagrams; Boussinesq and FEM; have almost the same pattern especially for deeper depths (Z>2m) although FEM solution has higher values than Boussinesq solution. These differences can be because of different assumptions in two approaches here, e.g. in Boussinesq formulation the soil is assumed to be linear elastic isotropic while in nature and in this study soil is assumed to be elasto-plastic. For a layered system, E₂/ E_1 =30, when the surface layer is weaker then vertical stresses in upper layer exceeds the Boussinesq values.

At the interface, the vertical stress goes down to less that 4kPa from almost 13kPa or dropped to less than 70% of its origin value. This means first layer transfers less vertical stress to the second layer. In other words, the effect of the strength of bottom layer has less contribution to stress distribution when the upper soil becomes weaker compared to the bottom layer. In addition, for deeper values and near to depth of 8 m all graphs tend to have the same value of 4 kPa.

10

5

кo

014

Figure 11 shows the effect of clay depth on stress distribution at four different values of $h_1/B = 0.15$, 0.5, 1 and 1.5. It can be seen that for smaller value of h_1/B , e.g., for the value of 0.15 stress is higher near earth surface: almost 550 kPa, and moving from soil surface toward soil depth leads to a decrease in stress: to 200 kPa at the depth of 8m. It should be noted for the depth of more than 6 metres h_1/B variation has small effect on vertical stress distribution and all graphs tend to have same values.

Shear strength effect of clay on failure mechanism development

In Figure 12 the results of plastic shear strain are plotted at failure point for smooth rigid footing for the case of clay-oversand. The interface of two layers is shown with a dark line in each figure. As a quick review, it is obvious that failure mechanism is deeper when h₁/B is higher. It should be noted that in the present method it is assumed that soil with elasto-plastic behaviour deforms under load while footing as a rigid body does not. In addition, soil element yields progressively in soil body from any element to the next element and a shear surface can be obtained as shown in Figure 12. It can be seen from Figure 12a when h, is smaller, the failure mechanism does not only shrink into the top layer.









In other words, for the smaller value of h_1/B plastic zone goes to bottom layer and does not stay only in top layer. With increasing h_1 , the plastic zone only stays in top layer and bottom layer's strength has no effect on bearing capacity value after specific depth. The plastic zone tends to stay in top layer as the height of weak soil increases and does not go to the stronger layer which aggresses well with results obtained by Potts et al. [16] and Zhu [2].

The summary of clay depth effect on plastic shear strain value, PE, is shown in Figure 13. It can be seen that the smallest thikckness of clay layer, $h_1/B=0.1$ has the maximum plastic shear strain of 1.5 %

and increasing clay depth to 0.5, 1 and 1.5 leads to a decrease in plastic shear strain going down to $0.9,\,0.54$ and 0.48~% .

Effect of material properties on magnitude and direction of displacement

Figure.14 illustrates displacement vectors at failure for clay-oversand and sandy soil in the case of smooth rigid footing under same magnitude downward displacement applying on both footings.



Figure 13: Effect of clay depth on shear strain distribution value





It can be seen that the displacement at top of the surface near the footing has a downward direction in sandy soil, Figure 14a, while in clayey soil displacement vectors are about 45° to the horizontal axis, Figure 14b. These results have a good agreement with those obtained by Ziaei et al. [9] and Potts [6]. It should be noted that in both cases same displacement value has been applied on top of soil.

In addition, after applying the same pressure on top of soil, 140 kPa, displacement under both footings is observed and the results are illustrated in Figure 15. It can be seen that under same pressure, maximum displacement in clayey soil is almost 50 times higher than those happen in sandy soil. In other words, in the case of $h_1/B=0.5$ the displacement under footing has a value of 0.2 m for clayey soil, Figure 15b, while under same pressure only 0.004 m displacement occurs in sandy soil, Figure 15a.

Conclusions

In this paper, a numerical analysis was carried out to investigate the influence of different parameters on ultimate bearing capacity of layered soil and on soil failure mechanism. The soil was modelled as an elasto- plastic material and computation were carried out using

FEM software, ABAQUS.

For homogenous soil profile, the effect of soil parameters such as dilation angel, footing roughness and initial condition was studied on soil behaviour. It is found that with increasing dilation angel the wider and the deeper failure mechanism is accrued under the footing. In addition the failure mechanism for rough interaction is deeper and wider than those for smooth ones. The initial condition, K_0 , has an effect on soil behaviour before failure but has no effect on bearing capacity value.

The bearing capacity value of one-layer sandy soil obtained through ABAQUS was compared with those predicted by Terzaghi's equation. It is concluded that for FEM analysis, the values for bearing capacity with considering dilation angel is 13% higher than those with no dilation and bearing capacity obtained by Terzaghi has the value between those two FEM values.

In two-layer-soil comparing to one-layer-soil, bearing capacity decreases dramatically to less than 70% of its value by adding the clay thickness of $h_1/B=0.15$ on top of sand. Increasing depth of clay leads to smaller values for bearing capacity showing that top layer mainly controls bearing capacity value.

According to Michalowski [18], a so-called critical depth in which strength of bottom layer has no influence on bearing capacity of whole model exists and this depth in this study is $h_{,/B=1.5}$ or $h_{,=4.5}$.

For smaller values of h_1 , the failure mechanism goes further to the bottom layer while with increasing thickness of clay, the plastic zone only shrinks into top layer.

Direction and magnitude of displacement vectors are much smaller and more downward when the top layer is sandy soil.

In the presented study the approach applied is straightforward for a two-layer soil and is applicable for multi-layer soil profiles as well. However, in this study only a short-term stability of footing was considered, the study on long-term behaviour of material would be of interest. In addition, the effect of footing-soil interaction can be taken into consideration in the future analysis.

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