

## Ideal Gas Law and the Greenhouse Effect

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### Abstract

This communication gives an explanation why the ideal gas law is sufficient to calculate the observed surface temperature of celestial bodies, if the mass density and the pressure of the respective atmospheres are known. The ideal gas law provides also a nice tool to check the experimental consistency of the parameters obtain by other measurements. But it does not permit to directly determine the radiative forcing.

**Keywords:** Earth; Planets; Thermodynamics; Atmosphere; Radiation

### Introduction

As shown by several recent papers, the median temperature of the earth and several planets and moons possessing an atmosphere can be explained with the ideal gas law [1-3]. Formally, this is not surprising, because the composition, the mass density and the pressure of the gases are well established by measurements. The ideal gas law links pressure,  $p$ , volume,  $V$ , and temperature,  $T$ , in the simple relation  $pV = RT$ . It is one of the fundamental laws of thermodynamics. The ideal gas constant is  $R = 8314 \text{ J K}^{-1} \text{ Mol}^{-1}$  (Mol in kg). The molar volume  $V$  can be replaced by the known mass density,  $\rho$ , and the molar mass,  $M$ , by  $V = M/\rho$ . We get

$$T = pM/(\rho R) \quad (1)$$

At sea level, the average pressure,  $p_0 \sim 1.014 \times 10^5 \text{ Pa}$ , the molar mass of the atmosphere  $\sim 29.0 \text{ kg}$  and the average mass density,  $\rho_0 \sim 1.225 \text{ kg m}^{-3}$ , yield the temperature,  $T_0 = 287.6 \text{ K}$ . This value agrees well with the generally accepted value inferred from satellite measurements. The question arises: "Why is the retention of the radiative energy playing no explicit role in this simple thermodynamic law?" In the following we are trying to give a simple answer based on physical arguments.

The earthly atmosphere is adiabatically compressed thanks to the gravitational field acting on it. This is manifested by the fact that the temperature of the troposphere decreases with the height above sea level,  $h$ . The pressure of the troposphere decreases approximately exponentially with the height above sea level,  $h$ , according to

$$p = p_0 \exp(-h/H) \quad (2)$$

where  $H \sim 8000 \text{ m}$ . This law is strictly valuable for an isothermal atmosphere. Thanks to gravity, the atmosphere is adiabatically compressed. This imposes the additional relation  $pV^k = \text{constant}$  that is equivalent to

$$p(M/\rho)^{k-1} = \text{constant} = C_0 \quad (3)$$

The exponent  $k$  is given by the ratio of the specific heat capacity at constant pressure divided by that of constant volume. For the atmosphere, we have  $k = 2.4$ . The constant  $C_0$  is arbitrary as it depends on the arbitrary choice of known data sets. With the Eqs. (1) - (3), we calculate the adiabatic temperature profile given by

$$T = T_0 \exp[1 - (h/H)(k-2)/(k-1)]. \quad (4)$$

The derivative of Eq. (4) evaluated at  $h=0$  yields the so-called temperature lapse rate given by

$$\Delta T/\Delta h = - (T_0/H)(k-2)/(k-1) = -0.0103 \text{ K/m} \quad (5)$$

The temperature lapse rate is in fair agreement with observations throughout the troposphere. It amounts to  $\sim 1\text{K}/100 \text{ m}$ . Day to day observations can strongly deviate due to weather conditions such as stratus (temperature inversion) or instable weather conditions. The temperature profile is not valid in the stratosphere and beyond, because of the strong influence of short wave electromagnetic and cosmic radiation. Strong deviation from theory is experimentally observed and justified for atmospheric pressures  $< 10^4 \text{ Pa}$ .

We now try to understand why  $T_0$  does not seem to depend on electromagnetic radiation. The index of refraction of the air at  $h=0$  is assumed to be continuous, with  $n = 1.000272$ . The mean distance,  $d$ , between the "air" molecule of the troposphere is approximated by

$$d = (V/L)^{1/3} = 3.34 \times 10^{-9} \text{ m} \quad (6)$$

where  $V = 22.4 \text{ m}^3$  and  $L = 6.022 \times 10^{26}$ , the constant of Avogadro per kg mol. By considering a small vacuum gap between the elementary volume given by  $V/L$ , we can estimate the radiation force,  $F_r$ , by:

$$F_r = 2SA(n^2 - 1)/c \quad (7)$$

with  $S \sim 1300 \text{ W m}^{-2}$  the radiation intensity;  $A \sim d^2$  the cross section of the elementary volume; and  $c$  the speed of light. We note that  $d$  is more than an order of magnitude larger than the covalent radius of air molecules. Accordingly, Eq. (7) overestimates the force on the molecule due to radiation pressure. This is to be compared to the gravitational force the "air" molecule experiences. With its mass,  $\mu = 4.85 \times 10^{-26} \text{ kg}$ ,  $\mu g$  becomes  $4.76 \times 10^{-25} \text{ N}$  with  $g = 9.81 \text{ ms}^{-2}$ . The relative force magnitude is:

$$|F_r/\mu g| < 0.11 \quad (8)$$

This ratio shows that the radiative force is small compared to the gravitational one. The ideal gas law thus needs no modification for this application. The agreement of the calculated temperature lapse rate with experiments, is a further indication that the ideal gas law is valid under the perturbations by gravity and radiation. The ideal gas law can thus be applied without any reservation.

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## Discussion

Independent calculations and observations of the surface temperatures given in the molar mass version of the ideal gas law points to a very low climate sensitivity and new insights on the physical nature of the atmospheric greenhouse effect deduced from an empirical planetary temperature model [1,2], agree for most planets and moons that have a substantial atmosphere. Only Mars is an exception thanks to its thin atmosphere ( $p < 10^4$  Pa). A fortuitous coincidence is therefore highly unlikely [3].

The ideal gas law is a pillar of thermodynamics for which we do not need to know the physical details. As  $RT$  is a measure of the stored thermal energy per mol, the gas law only describes a specified equilibrium condition. It does not give any details as to how the equilibrium is achieved nor to its contributing causes. Accordingly, the mean surface temperature,  $T_0$ , deduced from the gas law should be equal to the one determined by other means. Therefore, it does not give any new insight concerning the energy balance, but it gives  $T_0$  a real physical meaning [4,5].

Having this independent check of  $T_0$ , we have a very reliable starting point to estimate the heat contribution by the so-called greenhouse gases. It is noteworthy that the forcing calculations have been getting noticeably smaller by the years. The calculations by Reinhart 5 for  $\text{CO}_2$  have yielded the smallest contribution, despite his claim that his value represents an upper limit that cannot be exceeded. His calculations avoided the radiation balance by using a perturbation approach based on the observed  $T_0$  value. He maintains that a valid energy

balance necessitates the precise knowledge of the terrestrial emissivity, absorptivity and reflectivity and optical multi-scattering processes as well as nucleation data necessary to form clouds consisting of ice and/or water droplets. Most of these parameters are very poorly known. As an example, most of the Albedo data used is based on educated guesses.

## Conclusion

To conclude, the ideal gas law does not permit to estimate the “greenhouse” effect. But it puts the mean temperature concept on a solid footing for calculations of the radiation forcing. Only precise calculations based on  $T_0$  are capable to provide credible data of the radiation forcing. The author wants to thank Pierre Jacquot and Christophe de Reyff for finding these recent publications, for many interesting discussions, and for the critical reading of this manuscript.

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