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On Some Types of αrps-Closed Maps

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Abstract

This paper is continues to study of a new type of closed maps which is called αrps-closed map. As well as, we give and study other types of αrps-closed maps which are (αrps-closed maps, strongly αrps-closed maps and almost αrps-closed maps) in topological spaces. Also, we will study the relation between these mappings and discussion some properties of these maps.

Keywords: αrps-closed; Topology; Mappings; Subset

Introduction

Mappings play as important role, in the study of modern mathematics, especially in topology and functional analysis [1-5]. Different types of closed and open mappings were studied by various researchers [6]. Generalized closed mappings were introduce and studied. After him different mathematicians worked and studied on different versions of generalized maps [7].

Hammed introduced and studied αrps-closed sets and also introduce the notion (αrps-continuous, αrps-irresolute and strongly αrps-continuous) functions [8].

In this paper, we introduce and study new types of closed maps namely αrps-closed map in topological spaces and we use this maps to give other types of αrps-closed map which are (αrps-closed maps [9-13], strongly αrps-closed maps and almost αrps-closed maps) and we discussion the properties of these maps as well as, shows the relationships between some types of these maps [14-18].

Throughout this paper (X,τ), (Y,σ) and (Z,μ) (or simply X,Y and Z) represent non-empty topological spaces [19-22]. For a sub set A of a space X.cl (A), int (A) and A^c denoted the closure of A, the interior of A and the complement of A in X respectively [23].

Preliminaries

Some definition and basic concepts have been given in this section.

Definition

A subset A of a space X is said to be:

1. Semi-open [9] If A ⊆ cl(int t(A)) and semi-closed set if int(cL(A)) ⊆ A.
2. α-Open set [16] If A ⊆ int(cl(int(A))) and α-closed set if cl(int(cl(A)) ⊆ A.
3. Preopen set [15] If A ⊆ int(cl(A)) and preclosed if cl(int(A) ⊆ A.
4. Semi-preopen set [1] If A ⊆ cl(int(cl(A)) and semi-preclosed if int (Cl(int(A)) ⊆ A).
5. Regular open [20] If A=int(cl(A) and regular closed if A=cl(int(A)).
6. Regular α-open [21] if there is a regular open set U such that U ⊆ A ⊆ αcl(U).

The semi-closure (resp. α-closure, semi-pre closure), of a sub set A of X is the intersection of all semi-closed (resp. α-closed, semi-pre closed) sets containing A and denoted by scl(A) (resp. αcl(A), resp. spcl(A).

Remark: It has been proved that:

1. Every regular closed set and closed set in a space X is an αrps-closed set.
2. Every αrps-closed set is (sg-closed, gs-closed, αg-closed, gα-closed, rg-closed and rgα-closed) set.

Definition

A sub set A of a space X is said to be a:

1. Generalized closed set (briefly, g-closed) [10] if cl(A) ⊆ U whenever A ⊆ U and U is an open set in X.
2. Generalized semi-closed set (briefly, gs-closed) [3] if scl(A) ⊆ U whenever A ⊆ U and U is an open set in X.
3. Semi-generalized closed set (briefly, sg-closed) [5] if scl(A) ⊆ U whenever A ⊆ U and U is a semi-open set in X.
4. Generalized α-closed set (briefly, gα-closed) [13] if αcl(A) ⊆ U whenever A ⊆ U and U is an α-open set in X.
5. α-Generalized closed set (briefly, α g-closed) [12] if αcl(A) ⊆ U whenever A ⊆ U and U is an open set in X.
6. Regular generalized closed set (briefly, rg-closed) [18] if cl(A) ⊆ U whenever A ⊆ U and U is a regular open set X.
7. Regular generalized α-closed set (briefly, rgα-closed) [21] if cl(int(A)) ⊆ U whenever A ⊆ U and U is a regular α-open set.
8. Pre-semi closed] if spcl(A) ⊆ U whenever A ⊆ U and U is a g-open.
9. Regular pre-semi closed (briefly, rps-closed) [19] if spcl (A) ⊆ U whenever A ⊆ U and U is an rg-open set in X.

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10. αrps-Closed set [8] if αcl(A) ⊆ U whenever A ⊆ U and U is arps-open set in X.

The complement

g-Closed (resp. gs-closed, sg-closed, gα-closed, αg-closed, rg-closed, rgα-closed, rps-closed and αrps-closed) sets is called a g-open (resp. gs-open, sg-open, gα-open, αg-open, rg-open, rgα-open, rps-open and αrps-open) sets,

The class: The regular closed (resp. g-closed, sg-closed, gs-closed, αg-closed, gα-closed, rg-closed, rgα-closed, rps-closed and αrps-closed) subsets of X is denoted by RC(X,τ)[resp. GC(X,τ), SGC(X,τ), GSC(X,τ), αGC(X,τ), GαC (X,τ),RGC(X,τ), RGαC(X,τ), RPSC(X,τ) and αRPSC(X,τ)].

Definition

A map f: (X, τ) ⟶ (Y, σ) is called

1. **Closed map:** if f(A) is a closed set in (Y,σ), for every closed set A in (X, τ) [20].
2. **g-Closed map:** if f(A) is g-closed set in (Y,σ), for every closed set A in (X, τ) [11].
3. **sg-Closed map:** if f(A) is sg-closed set in (Y,σ), for every closed set A in (X, τ) [7].
4. **gs-Closed map:** if f(A) is gs-closed set in (Y, σ), for every closed set A in(X, τ) [7].
5. **gα-Closed map:** if f(A) is gα-closed set in (Y,σ), for every closed set A in (X, τ) [6].
6. **αg-Closed map:** if f(A) is αg-closed set in (Y, σ), for every closed set A in (X, τ) [6].
7. **rg-closed map:** if f(A) is rg-closed set in (Y, σ), for every closed set A in (X, τ) [4].
8. **rgα-Closedmap:** if f(A) is rgα-closed in (Y, σ), for every closed set A in (X, τ) [22].
9. **Almost-Closed map:** if f(A) is a closed set in (Y, σ), for every regular closed set A in (X, τ) [17].

Definition : A topological space X is said to be:

1. **$T^*_{1/2}$ - spaces:** If every rg-closed sets is closed [13].

2. **T_b-space**: If every gs-closed sets is closed [6].

3. **αT_b-space**: If every αg- closed sets is closed [6].

4. **$T_{1/2}$-space**: If every g- closed sets is closed [10].

5. **Locally Indiscrete space**: if every closed set is a regular closed [3].

Definition: A function f: (X, τ) →(Y, σ) is said to be:

1. **Continuous function:** If the inverse image of each open (closed) set in Y is an open(closed) set in X [9].
2. **αrps-Continuous:** If f^{-1}(A) is αrps- closed set in X for every closed set A in Y [8].
3. **αrps-Irresolute continuous:** If f^{-1}(A) is an αrps-closed set in X for every αrps-closed set A in Y [8].
4. **Stronglyαrps-continuous continuous:** If f^{-1}(A) is closed set in X for every αrps-closed set A in Y [8].

Proposition

1. If X is a $T_{1/2}$-space, then every g-closed set in X is αrps-closed.
2. If X is a T_b-space, then every αrps-closed set in X is g-closed.
3. If X is a $T^{*1/2}$-space, then every αrps-closed set in X is closed.

αrps-Closed Maps

In this section, we introduce a new type of closed sets namely αrps-closed maps in topological spaces and study some of their properties.

Definition : A map f: (X, τ)⟶(Y, σ) is called αrps-closed map if f(A) is αrps-closed set in (Y, σ), for every closed set A in (X, τ).

Proposition: Every closed map is αrps-closed map.

Proof : It follows from definition of closed map and fact that every closed set is αrps-closed map.

Remark: The converse of above proposition need not be true as seen from the following example.

Example: Let X={a, b, c} with the topology τ={X, θ, {a}, {a, c}}, where αRPSC(X, τ)={X, θ, {b}, {c}, {b, c}} and define f: (X, τ)→(X, τ) by f(a)=a, f(b)=c and f(c)=b, then f is αrps-closed map, but f is not closed map, since for the closed set A={b} in {X, τ}, but f(A)=f({b})={c} is αrps-closed set in (X, τ), but is not closed set in (X, τ).

Proposition : Every αrps-closed map f : (X, τ) →(Y, σ) is

1. αg-Closed map.
2. gα-Closed map.
3. sg-Closed map.
4. gs-Closed map.
5. rg- Closed map.
6. rgα-Closed map.

Proof: 1: Let A be a closed set in (X, τ), since f is αrps-closed map. Thus f(A) is αrps-closed set in (Y, σ) and by using remark [every αrps-closed set is αg-closed set] we get f(A) is αg-closed set in (Y, σ). Hence, f: (X, τ) →(Y, σ) is αg-closed map. **2:** Let A be a closed set in (X, τ), since f is αrps-closed map. Thus f(A) is αrps-closed set in (Y, σ) and by using remark, [every αrps-closed set is gα-closed set] we get f(A) is gα-closed set in (Y, σ). Hence, f: (X, τ) →(Y, σ) is gα-closed map.

The proof of steps 3, 4, 5, and 6 are similar to step 1 and 2.

The following example show the converse of proposition need not be true in general.

Example : Let X={a, b, c}, with the topology τ={X, θ, {a}, {b, c}}, where αRPSC(X, τ)={X, θ, {a}, {b, c}}, GαC(X,τ) =SGC(X, τ)=GSC(X, τ)=RGC(X, τ) =RGαC(X, τ)={X, θ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}, define f: (X, τ) →(X, τ) by f(a)=b, f(b)=a and f(c)=c, then it is clear that f is (αg-closed, gα-closed, sg-closed, gsclosed, rg-closed and rgα-closed) map, but f is not αrps-closed, since for the closed set A={b, c} in {X, τ}, but f(A)=f({b, c})={a,c} is (αg-closed, gα-closed, sg-closed, gs-closed, rg-closed and rgα-closed) set in (X, τ), but is not αrps-closed set in (X, τ).

Remark: The concepts of g-closed map and almost closed map are independent to αrps-closed map. As show in the following examples.

Example

Let X={a, b, c} with the topology τ={X, θ, {a}, {a, c}}, where αRPSC(X, τ)={X, θ {b},{c}},{b, c}}, GC(X,τ)={X,θ,{b},{a, b},{b, c} and define f: (X, τ) →(X, τ) by f(a)=a, f(b)=c and f(c)=b, then f is αrps closed map, but f is not g-closed map, since for the closed set A={b} in {X, τ}, but f(A)=f({b})={c} is αrps-closed set in (X, τ), but is not g-closed set in (X, τ).

Example

Let X={a, b, c}, with the topology τ={X, θ, {a}, {b, c}}, where αRPSC(X, τ)={ X, θ, {a}, {b, c} }, GC(X, τ)={X, θ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c} }. Define f: (X, τ) →(X, τ) by f(a)=b, f(b)=a and f(c)=c, then it is clear that f is a g-closed map, but f is not αrps–closed map, since for the closed set A={a} in {X, τ}, but f(A)=f({a})={b} g-closed set in (X, τ), but is not αrps-closed set in (X, τ).

Example

Let X={a, b, c} with the topology τ={X, θ, {a}}, where αRPSC(X, τ)={X, θ {b},{c},{b,c}}, and the set of regular closed set={X,θ} define f: (X, τ) →(X, τ) by f(a)=b, f(b)=a and f(c)=b, then f is almost -closed map, but f is not αrps-closed map, since for the closed set A={b, c} in {X, τ}, but f(A)=f({b, c})={a, c} is not αrps-closed set in (X, τ).

Example

Let X =Y={a, b, c} with the topologies τ={X, θ, {a}, {b}, {a, b} } and σ={Y,θ,{a},{a,c}}, where αRPSC(Y,σ)={Y, θ, {b}, {c}, {b, c}}, and RC(X, τ)={X,θ}, {a, c}, {b, c}}, let f: (X, τ)→(Y, σ) be a map defined by f(a)=f(c)=c and f(b)=b, then f is αrps-closed map, but f is not almost -closed map, since for the regular closed set A={a, c} in {X, τ}, but f(A)=f({a, c})={c} is not closed set in (Y, σ).

The following propositions give the condition to make the propositions and remark are true:

Proposition

If f: (X, τ) →(Y, σ) is αrps-closed map and Y is a$T^{*1/2}$ - space, then f is a closed map.

Proof: Let A be a closed set in (X,τ), since f is αrps-closed map. Thus f(A) is αrps-closed set in (Y, σ) and by using remark [every αrps-closed set is rg-closed set] we get, f(A) is rg-closed set in (Y, σ). Also, since Y is a$T^{*1/2}$-space then, by definition. we get f(A) is a closed set in (Y, σ).

Hence, f is an αrps-closed map

Proposition: A map f : (X, τ) →(Y, σ)is αrps-closed map, if f is a

1. αg-Closed map and Y is αT_b-space.
2. gα-Closed map and Y is αT_b-space.
3. gs-Closed map and Y is T_b-space.
4. sg-Closed map and Y is T_b-space.
5. rg-Closed map and Y is$T^{*1/2}$-space.
6. rgα-Closed map and Y is$T^{*1/2}$-space.

Proof: 1. Let A be a closed set in (X, τ), since f is αg-closed map. Thus f(A) is αgclosed set in (Y, σ), by hypotheses Y is a αT_b –space, then by definition, we get, f(A) is a closed set in (Y, σ), and by remark, every closed is αrps-closed set), hence f(A) is αrps-closed set in (Y, σ). Therefore, f is αrps-closed map.

2. It is follows from the fact (every gα-closed map is an αg-closed map [6]) and since Y is a αT_b –space, then by we get, f is αrps-closed.

3. Let A be a closed set in (X, τ), since f is gs-closed map. Thus, f(A) is gs-closed set in (Y, σ), also since Y is a T_b-space, then f(A) is a closed set in (Y, σ), and by remark (every closed set is αrps-closed set), hence f(A) is αrps-closed set in (Y, σ) Therefore, f is αrps-closed map.

4. It is follows from the fact (every sg-closed map is an gs-closed map) and since Y is a T_b – space, then by we get, f is αrps-closed.

5. Let A be a closed set in (X, τ), since f is rg-closed map. Thus, f(A) is rg-closed set in (Y, σ), also since Y is a $T^{*1/2}$-space, then by definition. We get f(A) is a closed set in (Y,σ), and by remark (every closed set is αrps-closed set), hence f(A) is αrps-closed set in (Y, σ) Therefore, f is αrps-closed map.

6. It is follows from the fact (every rgα-closed map is an rg-closed map), and since Y is a $T^{*1/2}$-space, then by we get, f is αrps-closed.

Similarly, we proof the following proposition.

Proposition

1. If f: (X, τ) →(Y, σ) is g-closed map Y is $T_{1/2}$-space, then f is a αrps closed map.
2. If f: (X, τ) →(Y, σ) is αrps-closed map Y is$T^{*1/2}$-space, then f is a g closed map.
3. If f: (X, τ) →(Y, σ) is αrps-closed map Y is $T^{*1/2}$-space, then f is almost-closed map.
4. If f: (X, τ) →(Y, σ) is almost-closed map and (X, τ) is a locally indiscrete then f is αrps-closed map.

Remark: The composition of two αrps-closed maps need not beαrps-closed map in general, the following example to show that.

Example

Let X=Y=Z={a, b, c} with the topologies τ={X, θ, {a}, {a, c}}, σ={y, θ,{a}},μ={Z,θ,{a},{b},{a,b}} where αRPSC(Y,σ)={Y, θ, {b},{c},{b, c}} and αRPSC(Z, μ)={Z, θ, {c}{a, c}, {b, c}}. let f: (X, τ)→(Y, σ) be the identity map, and g: (Y,σ)→(Z, μ) be a map defined by g(a)=b and g(b)=a and g(c)=c, then it is easy to see that f and g are αrps-closed map, but g of: (X,τ)→(Z,μ) is not αrps-closed map,since for the closed set A={b} in {X, τ}.

g of (A)=g of ({b})=g(f({b})=g({b})={a}, which is not αrps-closed set in (Z, μ). Hence, f is not αrps-closed map.

The following proposition gives the condition to make the composition two αrps-closed maps is also αrps-closed map.

Proposition

Let f: (X, τ) →(Y, σ) and g: (Y, σ)→(Z,μ) be two αrps-closed maps and Y is a $T^{*1/2}$-space, then g of (X,τ) →(Z,μ) is αrps-closed map.

Proof: Let A be a closed set in (X, τ), Thus f(A) is αrps- closed set in (Y, σ), since Y is a $T^{*1/2}$-space, then by proposition. we get f(A) is a closed set in (Y, σ), also, since g is αrps-closed map, hence g(f({A}) is aαrps-closed set in (Z,μ). But g(f{A})=g of (A), that is (f{A})=g of is aαrps-closed set in (Z,μ). Therefore g (X, τ) →(Z,μ), is αrps-closed map.

Proposition: If f: (X, τ) →(Y, σ) is a closed map and g: (Y, σ) →(Z, μ) is an αrps- closed map, then g of (X,τ)→(Z,μ) is αrps-closed map.

Proof: Let A be a closed set in (X, τ), Thus f(A) is a closed set in (Y,σ), since g is αrps-closedmap, hence g(f({A}) is an αrps-closed set in (Z,μ). That is g(f({A})=g of (A) is aαrps-closed set in (Z,μ). Therefore, g of: (X,τ)→(Z,μ) is αrps-closed map.

Remark

If f: (X, τ) →(Y, σ) is αrps-closed map and g: (Y, σ) →(Z, μ) is a closed map, then g of (X,τ)→(Z,μ) need not be αrps-closed map, and this is shown by the following example:

Example

Let X=Y=Z={a, b, c} with the topologies τ={X, θ, {a}, {a, c}}, σ={Y, θ,{a}}, μ={Z ,θ,{a},{b, c}} where αRPSC (Y,σ)={Y, θ, {b},{c}, {b, c}} and αRPSC(Z, μ)={Z, θ, {a}, {a, c}.let f:(X,τ)→(Y, σ), and g:(Y,σ)→(Z,μ) be two the identity maps then it is easy to see that f isαrps-closed map and g is a closed map, but g of: (X,τ)→(Z,μ)) is not αrps-closed map, since for the closed set A={b} in{X, τ}, then g of (A)=g of ({b})=g(f ({b})=g({b})={b}, which is not αrps-closed set in (Z,μ). Therefore, g of (X,τ)→(Z,μ) is not αrps-closed map.

The following propositions give the condition to make remark true:

Proposition

If f(X,τ)→(Y, σ) is an αrps- closed map, g:(Y, σ)→(Z, μ) is a closed map and let Y be $T^{*1/2}$-space, then g of: (X,τ) →(Z, μ) is αrps-closed map

Proof

Let A be a closed set in (X, τ), Thus f(A) is αrps- closed set in (Y, σ), since Y is a $T^{*1/2}$space, then by proposition. we get f(A) is a closed set in (Y,σ), also, since g is a closed map, hence g(f({A}) is a closed set in (Z, μ), by remark. [∀ closed set is] αrps-closed set]. Then, g(f({A}) is an αrps-closed setin (Z,μ). But, g(f({A})=g of (A), that is g of (A)is aαrps-closed set in (Z, μ). Therefore, g of: (X,τ)→(Z,μ) is αrps-closed map.

Proposition

Let f: (X, τ) →(Y, σ) and g: (Y,σ)→(Z,μ) be two maps, such that their composition g of (X,τ)→(Z,μ) is αrps-closed map,

1. If f is a continuous and subjective, then g is αrps-closed map.
2. If g is αrps* - continuous and injective, then f is αrps-closed map.

Proof: (i):- Let A be a closed set of (Y, σ), since f is a continuous, then f^{-1}(A)) is a closed set in (X,τ). Also, since g of: (X,τ)→(Z,μ) is an αrps-closed map, thus g of (f^{-1}(A)) is αrps-closed set in (Z,μ). That is g of (f^{-1}(A))=g(f(f^{-1}(A))=g(A), hence g(A) isαrps-closed set in (Z,μ). Since, f is surjective. Therefore, g is αrps-closed map.

Proof: (ii):- Let E be a closed set in (X,τ). Since g of (X,τ)→(Z,μ) is an αrps-closed map, thus g of (E)is an αrps-closed set in (Z,μ), since g is αrps*-continuous. Then g^{-1}(g of (E))=f(E) is an αrps-closed setin (Y, σ). Also, since f is injective. Therefore, f is αrps-closed map.

Some Types of αrps-Closed Maps

Some other types of αrps-closed maps are given in this section such as [α^*rps-closed maps, strongly αrps-closed maps and almost αrps-closed maps) with study the relationships between these types of maps.

Definition

A map f: (X, τ) ⟶ (Y, σ) is called α*rps-closed map if f(A) is αrps-closed set in (Y, σ), for every αrps-closed set A in (X, τ).

Proposition

Every α*rps-closed map is αrps-closed map.

Proof: Let f: (X, τ)⟶(Y,σ) be α^*rps-closed map and let A be a closed set in (X, τ), by remark ∀ closed set is a αrps-closed set]. Thus, A is aαrps-closed set in (X, τ). Since f is a α^*rps-closed map. Then, f(A) is a αrps-closed set in (Y, σ). Therefore, f is αrps-closed map.

Corollary : Everyα*rps-closed map is

1. αg-Closed map.
2. gα-Closed map.
3. sg-Closed map.
4. gs-Closed map.
5. rg-Closed map.
6. rgα-Closed map.

Proof

It is follows from proposition.

Remark: The converse of proposition are not true in general. It is easy to see that in example, f isα*rps-closed map, but is not closed, and in example it is clear that f is (αg-closed map, gαclosed map, sg-closed map, gs-closed map, rg-closed map and rgα-closed map), but is not α*rps-closed map.

1. The concepts of closed map and almost-closed map are independent to α*rps-closed map. It is clear see that in examples.

The following propositions give the condition to make the proposition, corollary and Remark are true.

Proposition: Let f: (X,τ)⟶(Y, σ) be an α*rps-closed map and Y is $T^{*1/2}$space then f is a

1. Closed map.
2. Almost-closed map.

Proof (i):- It is follows from proposition, we get f is a closed map.

Proof (ii): It is follows from the fact (∀ closed map is almost-closed map [17].

Proposition: Let f: (X, τ)⟶ (Y, σ) be any map, then f is α*rps-closed map, if X is $T^{*1/2}$-space and f is a

1. αg-closed map and Y is a αT_b-space.
2. gα-closed map and Y is a αT_b-space.

Proof (i)

Let A be an α*rps-closed set in (X, τ), since X is a $T^{*1/2}$-space, then by using proposition. We get, A is a closed set in X. Also, since f is αg-closed map. Thus, f(A) is αg-closed in (Y,σ) and since Y is αT_b-space, then f(A) is closed set in (Y, σ), by remark [∀-closed set is an α*rps-closed set]. Hence, f(A) is α*rps- closed set in (Y, σ). Therefore, f is α*rps-closed map.

Proof (ii)

It is follows from the fact (∀ gα-closed map is αg-closed map [6]), and Similarly, we proof the following proposition.

Proposition

Let f: (X, τ)⟶ (Y, σ) be any map, then f is α*rps-closed map, if X is $T^{*1/2}$-space and f is a

1. gs-Closed map and Y is a T_b-space.

2. sg-Closed map and Y is a T_b -space.
3. rg-Closed map and Y is $T^{*1/2}$-space.
4. rgα-closed map and Y is $T^{*1/2}$-space
5. Closed map.
6. αrps-Closed map.
7. g-Closed map.

Proposition

If f: (X, τ) ⟶ (Y, σ) is a almost - closed map and X is a $T^{*1/2}$-space and locally indiscrete, then f is α*rps-closed map.

Proof

Let A is a αrps-closed set in (X, τ). Since X is a $T^{*1/2}$-space, then by using proposition. We get A is a closed set in X. Also, since X is a locally indiscrete, then by definition of locally indiscrete we have, A is a regular closed set in X. since f is a almost-closed map. Thus, f(A) is a closed set in Y and by remark [∀ closed set in X]. Hence, f(A) is a αrps -closed set in Y. Therefore, f is α*rps- closed map.

Proposition

The composition of two α*rps-closed maps is alsoα*rps closed map.

Proof: Let f: (X, τ) ⟶ (Y, σ) and g: (Y,σ)→(Z,μ) be two α*rps -closed map, and A be αrps-closed set in X, since f is α*rps-closed map, then f(A) is an αrps - closed set in (Y,σ). Also, since g is an α*rps-closed map. Thus, g(f({A}) is aαrps-closed set in (Z,μ). That is g(f ({A})=g of (A)is a αrps- closed set in (Z, μ).

Therefore, g of (X,τ)→(Z,μ) is α*rps-closed map.

Proposition

If f:(X,τ)⟶(Y,σ) is αrps-closed map and g:(Y,σ) →(Z, μ) is α*rps-closed map, then g of: (X,τ)→(Z,μ) is αrps-closed map.

Proof: Let A be a closed set in (X, τ), then f(A) is αrps - closed set in (Y,σ). Also, since g is α*rps-closed map. Thus, g(f({A}) is aαrps-closed set in (Z,μ). That is, g(f({A})=g of (A)is aαrps-closed set in (Z, μ). Therefore, g of: (X,τ)→(Z,μ) is αrps-closed map.

Similarly, we proof the following corollary.

Corollary

If f:(X, τ)⟶(Y,σ) is a closed map and g:(Y, σ →(Z, μ) is α*rps -closed map, then g of (X,τ)→(Z,μ) is αrps-closed map.

Now, we give another type of αrps-closed map is called strongly αrps-closed map.

Definition

A map f: (X, τ) ⟶ (Y, σ) is called strongly αrps-closed map if f(A) is closed set in (Y, σ), for every αrps-closed set A in (X, τ).

Proposition

Every strongly αrps- closed map f: (X, τ)⟶(Y, σ) is

i. Closed map.
ii. Almost-closed map.
iii. g-Closed map.
iv. αrps-Closed map.
v. α*rps-Closed map.

Proof

i. Let A be a closed set in (X, τ), by using remark, step [∀ closed set is an αrps-closed]we get, A is an αrps-closed set in (X, τ). Since f is strongly αrps- closed map. Thus, f(A) is a closed set in(Y, σ). Therefore, f is a closed map.
ii. It is clear that from step [∀ strongly αrps-closed map is a closed and the fact (closed map is almost closed, [17].
iii. It is clear that from step [∀ strongly αrps-closed map is a closed and the fact (∀ closed map is g-closed, [4])
iv. It is clear that from step and the proposition.
v. Let A be an αrps-closed set in (X, τ). Since f is strongly αrps-closed map.

Thus, f(A) is a closed set in (Y, σ), by using remark, [∀ closed set is an αrps-closed set], then A is an αrps-closed set in (Y, σ). Therefore f is a α*rps-closed

Corollary: Every strongly αrps-closed map f: (X, τ) ⟶ (Y, σ) is

1. αg-Closed map.
2. gα-Closed map.
3. sg-Closed map.
4. gs-Closed map.
5. rg-Closed map.
6. rgα-Closed map.

Proof

It is clear that from proposition. The following examples show the converse of above proposition and corollary need not be true in general.

Example

Let X={a, b, c} with the topology τ={X, θ, {a}} and let f: (X, τ) →(X, τ) be an identity map. Then, it is clear that f is a closed map, [almost-closed map and g-closed map] but is not strongly αrps-closed, since for closed set A={b}, f(A)=f({b})={b} is not closed set in (X,τ).

Example

Let X=Y={a, b, c} with the topologies τ={X, θ, {a}}, and σ={ Y, θ, {a}, {a, c}}, where αRPSC (X, τ)={X, θ, {b},{c}, {b, c}} and let f: (X, τ) →(Y, σ) be an identity map. Then, it is clear that f is αrps-closed map and α*rps -closed map but is not strongly αrps-closed map, since for closed set A={c}, then f(A)=f({c})={c} is not closed set in (Y, σ).

Example

LetX=Y={a, b, c} with the topologies τ={X, θ, {a}}, and σ={Y, θ, {a}, {b, c} }. Define f: (X, τ) →(Y, σ) by f(a)=b, f(b)=a and f(c)=c. Then, it is clear that f is αg-closed map (gα-closed map, sg-closed map, gs-closed map, rg-closed map and rgα-closed map), but is not strongly αrps-closed map, since for closed set A={c}, then f(A)=f({c})={c} is not closed set in (Y,σ).

The following condition to make proposition and corollary are true

Proposition

Let f : (X, τ) →(Y, σ) be any map, then f is a strongly αrps- closed map if X is a $T^{*1/2}$-space and

1. Closed map
2. Almost- closed map.
3. g-Closed map.
4. gα-Closed map.
5. αg-Closed map.
6. rg-Closed map.
7. rgα-Closed map.
8. αrps-closed map.

Proof

It is follows from proposition and step and proposition.

Proposition

If f: (X, τ) →(Y, σ)is an α*rps-closed map and Yis a $T^{*1/2}$-space, then f is a strongly αrps-closed map.

Proof

Let A be an αrps-closed set in (X, τ). Since f is $α^*$rps-closed map. Thus f(A) is αrps-closed set in (Y, σ). Also, since Y is $T^{*1/2}$-space, then f(A) is a closed set in (Y, σ). Therefore, f is strongly αrps- closed map.

Proposition

If f: (X, τ) →(Y, σ) is almost-closed map and Xis a locally - indiscrete and $T^{*1/2}$-space, then f is a strongly αrps- closed map.

Proof

Let A be an αrps - closed set in (X, τ). Since X is a $T^{*1/2}$-space. Then, A isa closed set in (X, τ), also since X is a locally-indiscrete, thus A is a regular closed set in (X, τ), hence f(A) is a αrps-closed set in(Y, σ). Therefore, f is a strongly αrps-closed map.

Next, we give some proposition and results about the composition of strongly αrps-closed map.

Proposition

The composition of two strongly αrps-closed maps is also strongly αrps-closed map.

Proof

Let f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) →(Z, μ) be two strongly αrps-closed map, and A be αrps-closed set in X, since f is strongly αrps-closed map, then f(A) is a closed set in (Y, σ), by remark [∀ closed set is an αrps-closed set] Thus, f(A) is an αrps-closed set in (Y,σ). Also, since g is an strongly αrps closed map. Thus, g(f({A}) is a closed set in (Z,μ). That is g(f({A})=g of (A)is aαrps-closed set in (Z, μ). Therefore, g of: (X,τ)→(z,μ) is strongly αrps-closed map.

Similarly, we proof the following proposition.

Proposition

1. If f: (X, τ) ⟶ (Y, σ) is a strongly αrps-closed map and g: (Y, σ) →(Z, μ) is closed map, then g of (X,τ)→(Z, μ) is a strongly αrps-closed map.
2. If f: (X, τ) ⟶ (Y, σ) is a α*rps - closed map and g: (Y, σ) →(Z, μ) is strongly αrps-closed map, then g of (X,τ) →(Z, μ) is a strongly αrps-closed map.

Proposition

Let f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) →(Z, μ) be two any maps, then g of(X,τ) →(Z, μ) is a α*rps -closed map, if f is strongly αrps-closed map and

i. g- is αrps-closed map.

ii. g- is α*rps -closed map.

Proof

(i) Let A be αrps-closed set in X, since f is strongly αrps-closed map, then f(A) is a closed set in (Y, σ). Also, since g is αrps-closed map. Thus, g(f({A}) is a αrps-closed set in (Z,μ). That is g(f({A})=g of (A) is a αrps-closed set in (Z, μ). Therefore, g of: (X,τ)→(Z, μ) is α*rps -closed map.

The proof of steps.

Remark

In the proposition the composition g of: (X,τ)→(Z, μ) need not be in general strongly αrps-closed map. As shows in the following example:

Example

Let X=Y=Z={a, b, c} with the topologies τ={X, θ, {a}, {b, c}}, σ ={Y,θ,{a}, {a, c}},μ={z,θ,{a}}. Let f: (X, τ) →(Y, σ) by f(a)=f(b)=b and f(c)=c and g: (Y,σ)→(Z,μ) be an identity map, then it is easy to see that f is strongly αrps-closed map and g is $α^*$rps- closed map, but g of (X,τ)→(Z,μ).

Is not strongly αrps-closed map,since for the closed set A={a}in {X, τ}, then g of (A)=g of ({a})=g(f({a})=g({b})={b}, which is not closed set in (Z,μ).

Proposition

Let f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) →(Z, μ) be two any maps then g of: (X,τ)→(Z, μ) is αrps-closed map, if g is strongly αrps-closed and

i. f is a closed map.

ii. f is a αrps-closed map.

Proof

(i) Let A be a closed set in X, since f is closed map, then f(A) is a closed set in (Y, σ), by remark [∀ closed set is an αrps-closed set, so we get f(A) is an αrps-closed set in(Y, σ). Also, since g is strongly αrps-closed map. Thus, g(f({A}) is a closed set in (Z, μ). That is g(f({A})=g of (A) is a closed set in (Z,μ) and by remark [∀ closed set is an αrps-closed set], so we get g of (A) is a closed set in (Z, μ). Therefore, g of: (X,τ)→(Z, μ) is α*rps-closed map. The proof of steps.

Remark

In the proposition the composition g of: (X,τ)→(Z, μ) need not be in general strongly αrps-closed map. As shows in the following example.

Example

Let X=Y=Z={a, b, c} with the topologies τ={X,θ,{a}}, σ={Y, θ,{a},{b},{a,b}},μ={Z,θ,{a},{a,c}} and let f: (X, τ)→(Y, σ) be an identity map and g: (Y,σ)→(Z,μ) be a mapping defined by g(a)=g(b)=c and g(c)=b, then it is easy to see that f is a closed map and αrps-closed map and g is strongly αrps- closed map, but g of: (X,τ)→(Z,μ)) is not strongly αrps-closed map, since for the closed set A={b}} in {X, τ}, then g of (A)=g of (A)=g of ({b})=g(f({b})=g({b})={c}, which is not closed set in(Z, μ).

The following proposition give the condition to make Remark true:

Proposition

Let f:(X, τ) ⟶ (Y, σ) and g: (Y, σ) →(Z, μ) be two any maps, then g of (X,τ) →(Z, μ) is a strongly αrps-closed map, if f is strongly αrps-closed map and (Z, μ) is a $T^{*1/2}$-space

1. g is αrps-closed map.
2. g is α*rps-closed map.

Proof

Let A be a αrps-closed set in X, then f(A) is a closed set in (Y, σ), by

Remark: [∀ closed set is an αrps-closed set], since g is αrps-closed map. Thus g(f({A}) is aαrps-closed set in (Z, μ). That is g(f({A})=g of (A) is an αrps-closed set in (Z, μ). Also, since Z is $T^{*1/2}$-space, so we get g of (A) is closed set in (Z, μ).Therefore, g of: (X,τ)→(Z, μ) is stronglyαrps-closed map.

Proposition

Let f:(X, τ) ⟶ (Y, σ) and g: (Y, σ) →(Z, μ) be two any maps, then g of :(X,τ) →(Z, μ) is strongly αrps-closed map, if g is strongly αrps-closed map, X is a $T^{*1/2}$-space and

1. f is a closed map
2. f is a αrps-closed map.

Proof

(i) Let A be αrps-closed set in X, since X is a $T^{*1/2}$-space, then by using proposition we get A is a closed set in X. Thus, f(A) is a closed set in Y, by remark, if (A) is an αrps-closed set inY. Also, since g is strongly αrps-closed map. Thus, g(f({A}) is a closed set in Z. That is g(f({A})=g of (A) is a closed set in Z. Hence, g of (X,τ)→(Z, μ) is strongly αrps-closed map.

Proposition: Let f:(X, τ) ⟶(Y, σ) and g: (Y, σ) →(Z, μ) be two any maps

(i) if g of: (X,τ) →(Z, μ) is strongly αrps-closed map and f is continuous surjective map, then g is a αrps-closed map.

(ii) if g of: (X,τ) →(Z, μ) is strongly αrps-closed map and f is strongly αrps-continuous surjective map, then g is strongly αrps-closed map

Proof (i)

Let A be a closed set in Y, since f is continuous, then $f^{-1}(A)$ is a closed set in (X,τ), by Remark(2-2)[∀ closed set is an αrps-closed set], so we get $f^{-1}(A)$ is a αrps-closed in X. Also, since g of: (X,τ)→(Z,μ) is a strongly αrps-closed map, thus g of ($f^{-1}(A)$is a closed set in (Z,μ) and Remark(2-2)[∀closed set is anαrps-closed set], hence g of ($f^{-1}(A)$ is an αrps-closed set in (Z, μ). That is g of ($f^{-1}(A)$)=g(f($f^{-1}(A)$=g(A), hence g(A) isαrps-closed set in (Z, μ) since, f is subjective. Therefore, g is αrps-closed map.

Proof (ii)

Let A be a αrps-closed set in Y. Since f is strongly αrps-continuous. Then, $f^{-1}(A)$ is a closed set in X, by Remark [∀ closed set is an αrps-closed set], so we get $f^{-1}(A)$ is a αrps-closed in X. Also, since g of (X,τ)→(Z, μ) is strongly αrps-closed map, hence map, hence g of ($f^{-1}(A)$ is a closed set in (Z, μ), since, f is subjective. That is g of ($f^{-1}(A)$)=g(f($f^{-1}(A)$)=g(A), hence g(A) is closed set in Z. Therefore, g is strongly αrps-closed map.

In the following, we give other type of αrps - closed maps which is called almost αrps - closed map.

Definition

A map f: (X, τ) ⟶ (Y, σ) is called almostαrps-closed map if f(A) is αrps-closed set in (Y, σ), for every regular closed set A in(X, τ).

Proposition

Every almost closed map is almost αrps- closed map.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a almost closed map and A be a regular closed set in (X, τ). Then, f(A) is a closed set in (Y, σ) and by using remark we get f(A) is an αrps-closed set in (Y, σ). Hence, f is almost αrps-closed map.

Proposition

Everyαrps-closed map is almost αrps-closed map.

Proof: Let f: (X,τ)⟶ (Y,σ) be an αrps-closed map and A be a regular closed set in (X,τ). Since (∀ regular closed [3]). Then, A is a closed set in X.

Thus, f(A) is an αrps-closed set in (Y, σ). Hence, f is almost αrps-closed map.

Corollary

1. Every closed map is almostαrps-closed map.
2. Everyα*rps-closed map is almost αrps-closed map.
3. Every strongly αrps-closed map is almostαrps-closed map.

Proof: The converse of proposition and corollary need not be true in general.

Example

Let X=Y={a, b, c} with the topologies τ={X, θ, {a}} and σ={Y, θ, {a}, {b}, {a, b}}, where αRPSC(X,τ)={X, θ, {b}, {c}, {b, c}}RC(X,τ)={X,θ} and αRPSC(Y, σ)={Y, θ, {c}, {a, c}, {b, c}}. Define f:(X, τ)→(Y, σ)by f(a)=c f(b)=a and f(c)=b. Then, it is clear that f is almost αrps-closed map but is not closed map (αrps-closed, α*rps-closed and strongly αrps-closed) map, since for closed set A={b, c} in (X, τ), f(A)=f({b, c})={a, b} is not closed and (αrps-closed)set in Y.

Example

Let X=Y={a, b, c} with the topologies τ={X, θ, {a}} and σ={Y, θ, {a}, {a, c}}, where RC(X, τ)={X, θ, {b, c}} and αRPSC(Y, σ)={Y, θ, {b}, {c}, {b, c}}.

Define f:(X, τ)→(Y, σ) by f(a)=b, f(b)=f(c)=c. Then, it is clear that f is almost αrps-closed map but is not almost closed map, since for regular closed set A={b, c} in (X, τ), f(A)=f({b, c})={c} is not closed in (Y, σ).

The following proposition give the condition to make, proposition and corollary are true:

Proposition

If f: (X, τ) →(Y, σ) is almost αrps-closed map and (Y, σ) is a $T^{*1/2}$-space, then f is a almost-closed set.

Proof: Let A be a regular closed set in (X, τ), since f is a almost αrps-closed map Then, f(A) is an αrps-closed set in Y. Also, since Y is a $T^{*1/2}$space, then by proposition. we get, f(A) is a closed set in Y. Hence, f is almost-closed map.

Proposition

If f: (X,τ)→(Y, σ) is almost αrps-closed map and X is a locally indiscrete space, then f is αrps-closed set.

Proof

Let A be a closed set in (X, τ), since X is a locally indiscrete, then by definition. We get, A is a regular closed set in X. Also, since f is a almost αrps-closed map. Then, f(A) is an αrps-closed set in (Y, σ). Therefore, f is αrps-closed map.

Proposition: Let f: (X, τ) →(Y, σ)be almost αrps- closed map and X be a locally indiscrete space and Y be a $T^{*1/2}$space, then f is a closed set.

Proof: Let A be a closed set in (X, τ), since X is a locally indiscrete, then by definition we get, A is a regular closed set in X. Also, since f is a almost αrps- closed map. Then, f(A) is an αrps-closed set in (Y, σ) and since Y is a $T^{*1/2}$space, then by proposition we get f(A) is a closed set in.

Proposition

Let f: (X,τ) →(Y, σ) be almost αrps-closed map and X be a locally indiscrete space and $T^{*1/2}$-space, then

1. f is aα*rps-closed set.
2. f is a strongly αrps-closed set if Y is a $T^{*1/2}$space.

Proof: (i) Let A be a αrps-closed set in (X, τ), since X is a $T^{*1/2}$space, then by proposition. we have, A is a closed set in X and since X is a locally indiscrete, then by definition. we get, A is a regular closed set in X. Also, since f is a almost αrps-closed map. Then, f(A) is an αrps-closed set in (Y, σ). Therefore, f is an α*rps -closed set.

Proof: (ii) Let A be a αrps-closed set in (X, τ), since X is a $T^{*1/2}$space, then by proposition. we have, A is a closed set in X and since X is a locally indiscrete, then by definition, we get, A is a regular closed set in X. Also, Thus, f(A) is an αrps-closed set in (Y, σ). Also, since Y is a $T^{*1/2}$space Hence, f(A) is a closed set in Y. Therefore, f is a strongly αrps-closed set.

Remark: The composition of two strongly αrps-closed maps need not be strongly αrps-closed map in general, the following example to show that.

Example: Let X={a, b, c, d }, Y=Z={a, b, c} with the topologies τ={X, θ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, { a, b, d }},σ={Y,θ,{a}},μ={Z,θ,{a},{b,c}} where RC(X, τ)={X,θ,{a,d},{b,c,d}}, RC(Y, σ)={Y, θ},RC(Z,θ){a},{b, c}}, αRPSC(Y, σ)={Y, θ, {b},{c}, {b, c}} and αRPSC(Z, μ)={Z, θ, {a}, {b, c}} Define f:(X, τ)→(Y, σ) by f(a)=f(d)=b, f(b)=f(c)=c and g: (Y,σ)→(Z,μ) be an identity map, then it is easy to see that f and g are almostαrps-closed map, but g of: (X,τ)→(z,μ)) is not almostαrps-closed map, since for the regular closed set A={a,d} in {X, τ} g of (A)=g of ({a,d})==g(f({a, d}))=g({b})={b}, which is not αrps-closed set in (Z,μ). Hence, g of is not strongly αrps-closed map the following proposition give the condition to make remark is true:

Proposition

If f:(X, τ)⟶(Y, σ) and g:(Y, σ)→(Z, μ) are two almost αrps-closed maps and Y is locally indiscrete and $T^{*1/2}$-space, then g of: (X,τ)→(Z, μ) is a almost αrps-closed map.

Proof: Let A be a regular closed set in X, thus f(A) is an αrps-closed set in Y. Since, Y is a $T^{*1/2}$space, then by proposition. we get f(A) is a closed set in Y. Also, since Y is a locally indiscrete, hence f(A) is a regular closed set in Y, since g is almost αrps-closed map. then g(f(A)) is a αrps-closed set in Z. But g(f(A))=g of (A). Therefore, g of: (X,τ) →(Z, μ) is a almost αrps-closed map. The proof of the following proposition it is easy.

Proposition

Let f:(X, τ)⟶(Y, σ) and g:(Y, σ)→(Z, μ) be two maps, then g of (X,τ)→(Z,μ) is a almost αrps-closed map, if f is almost αrps-closed and g is Figure 1.

(1) α*rps -closed map.

(2) Strongly αrps-closed map.

Remark: Here in the following diagram illustrates the relation between the **α**rps-closed mapping types (without using condition), where the converse is not necessarily true.

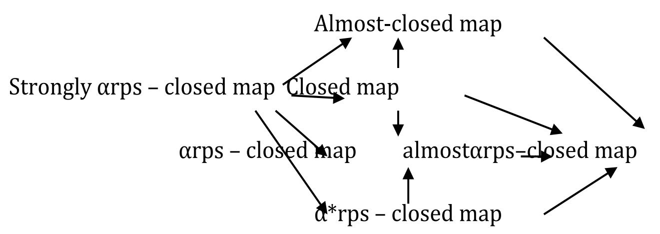

Figure 1: Illustrates the relation between the **α**rps- closed mapping types.

References

1. Andrijevic D (1986) Semi-preopen sets. MatVesnik 38: 24-32.
2. Andrijević D (1996) On b-open sets. Matematički Vesnik 205: 59-64.
3. Arya SP (1990) Characterizations of s-normal spaces. Indian J Pure Appl Math 21: 717-719.
4. Arockiarani I (1997) Studies on generalizations of generalized closed sets and maps in topological spaces.
5. Bhattacharyya P (1987) Semi-generalized closed sets in topology. Indian J Math 29: 375-382.
6. Devi R, Balachandran K, Maki H (1998) Generalized α-continuous maps and α-centralized closed maps. Indain J Pure Appl Math 29: 37-49.
7. Devi R (1993) Semi-generalized closed maps and generalized semi-closed maps. Mem Fac Sci Kochi Univ Ser A Math 14:41-54.
8. Hammed DM (2014) On Arps–Closed Sets in Topological Spaces. Engineering and Technology Journal 32: 271-286.
9. Levine N (1963) Semi-open sets and semi-continuity in topological spaces. Am Math Mon 70: 36-41.
10. Levine N (1970) Generalized closed sets in topology Rendiconti del Circolo Matematico di Palermo 19: 89-96.
11. Malghan SR (1982) Generalized closec maps. Pp: 82-88.
12. Maki H (1994) Associated topologies of generalized α-closed sets and α-generalized closed sets. Mem Fac Sci Kochi Univ 15: 51-63.
13. Maki H, Devi R, Balanchandran K (1993) Generalized α-closed sets intopology. Pp: 13-21.
14. Mashhour AS (1982) On preconlinuous and weak precontinuous mappings. In Proc Math Phys Soc Egypt 53: 47-53.
15. Nagaveni N (1999) Studies on generalizations of homeomorphisms in topological spaces. Ph.D Thesis.

16. Njåstad O (1965) On some classes of nearly open sets. Pacific Journal of Mathematics 15: 961-970.

17. Noiri T (1999) Almost αg-closed functions and separation axioms. Acta Mathematica Hungarica 82: 193-205.

18. Palaniappan N (1995) Regular generalized closed sets.

19. Mary TSI, Thangavelu P (2010) On regular pre-semi-closed sets in topological spaces. J Math Sci Comput Appl 1: 9-17.

20. Stone MH (1937) Applications of the theory of Boolean rings to general topology. Trans Am Math Soc 41: 375-481.

21. Vadivel A, Vairamanickam K (2009) rgα-closed sets and rgα-open sets in topological spaces. Int J Math Analysis 3: 1803-1819.

22. Vadivel A, Vairamanickam K (2010) rgα-Closed and rgα-open maps in topological spaces. J Math 10: 453-468.

23. Kumar MV (2002) Pre-semi-closed sets. Indian J Math 44: 165-181.