APPENDIX

Appendix A

A BRIEF DISCUSSION OF PHOTODETECTORS

In this part, we describe the distribution functions of light absorbed by the human retina photoreceptors called cones and probes. We focus on the cones.

Let $\mathbf{a_i(C)}$ represent the spectral color response absorbed by the retina. Three cones receptors have the spectra absorption distribution of $S_i(\lambda)$ i=1,2,3 respectively with certain overlapping between the absorption distribution functions, (particularly between S_1 and S_2 *i.e.* Red and Green). The visible color sensation is defined in the range:

$\lambda_{\min} \le \lambda \le \lambda_{\max}$ where $\lambda_{\min} \cong 380nm$ and $\lambda_{\max} \cong 780nm$,

and described by the spectral responses:

(A1) $$\mathbf{a_i(C)} = \int_0^\infty S_i(\lambda) C(\lambda) d\lambda$$

The retina cones visual spectra distributions of S_1,S_2 and S_3 decays outside the spectral bandwidth $[\lambda_{\min},\lambda_{\max}]$. From eq. (A1) we can learn that two spectral distribution $C(\lambda_1)$ and $C(\lambda_2)$ may produce identical spectral response $a_i(C_1)= a_i(C_2)$ for i=1,2,3. This means that two colors that look identical could have different spectral distribution.

In using primary colors, we observe from (A1) that the color matching coefficients h_{ij} are computed as follows:

(A2) $$\mathbf{h_{ij} = a_i(P_k)} = \int_0^\infty S_i(\lambda) P_k(\lambda) d\lambda \quad k=1,2,3$$

Where each primary color distribution (k=1,2,3) represents one of the basic colors (R, G or B) or any other suitable set of basic colors. In addition, we assume that each of the basic colors has a uniform energy distribution, *i.e.* the integral over the domain λmin to λmax is unity. Hence, a color matching procedure involves solving the following equations:

(A3) $$\sum_{k=1}^{3} \varepsilon_k h_{ik} = a_i(C) = \int_0^\infty S_i(\lambda) C(\lambda) d\lambda$$

This means, for three primary colors P_1,P_2 and P_3 and three types of the retina (cones) distribution functions $S_i(\lambda)$ the coefficient matrix is computed from the cross correlation distribution response given in (A1) and (A2). Given three numbers ε_k the linear combination set in (A3) describes the color perceived as $C(\lambda)$.

The idea of color matching in the biophotodetector array (retina cones) is somehow adopted to the physical photodetector array in the sense of the sensitivity to wavelength (*i.e.* Colors - Red, Green and Blue) and not to the array distribution, while in the retina it distributes non equispaced, in the CCD or CMOS photodetector array it distributes equispaced. Without loss of generality we assume that a photodetector array has a $D_i(x,y,\lambda)$ color distribution function (i=1,2,3) for the three photodetector array marked as R,G,B (could be any other finite set of colors or equivalent distribution functions). Let $I(\lambda,x,y)$ be the color intensity distribution object. The coordinate system (x,y) represents the photodetector coordinate system or the film coordinate system where every space point (X,Y,Z) represents the 3D real time manifold or real time scenario or environment is projected onto the film coordinate system having the following projection coordinate connections

(A4) $$\mathbf{x} = \xi \frac{\mathbf{X}}{\mathbf{Z}} \qquad \mathbf{y} = \xi \frac{\mathbf{Y}}{\mathbf{Z}}$$

where ξ represents the focal length of the film plane.

The photodetector array distribution response function is given mutatis mutandis to (A1) as follows:

(A5)

$$\sum_{k=1}^{3} v_k(x,y) q_{ik}(x,y) = g_i(x,y,I) = \int_0^\infty D_i(x,y,\lambda) I(x,y,\lambda) d\lambda$$

Where $g_i(x,y,I)$ represent the (x,y) photodetector cell response to the object I. Marking by R = $g_1(x,y,I)$, G = $g_2(x,y,I)$ and B = $g_3(x,y,I)$ the cell response to the object I respectively is D_1 for R, D_2 for G and D_3 for B of the photodectector array. The distribution functions decay outside a finite range, usually about 300nm to 800nm, for IR it is over 1200nm and in case of thermo it is around 4000nm.

Appendix B

Let

(1) $F(t) = P(t) \cdot G(t) + c \cdot P'(t)$ where c is constant

Define

(2) $\emptyset(t) = c \cdot e^{c^{-1} \int G(t)dt} = c \cdot e^{\frac{1}{c} \int G(t)dt}$

Then,

(3) $\frac{d}{dt}\left(\emptyset(t) \cdot P(t)\right) = \emptyset'(t) \cdot P(t) + \emptyset(t) \cdot P'(t) =$

(4) $e^{c^{-1} \int G(t)dt} \cdot \left(P(t) \cdot G(t) + c \cdot P'(t)\right) = e^{c^{-1} \int G(t)dt} \cdot F(t)$

Multiplying Eq. (1) by $\emptyset(t)$ and taking (3) and (4) under consideration we get

(5) $\emptyset(t) \cdot P(t) = \int e^{c^{-1} \int G(u)du} \cdot F(t) \cdot dt + L$, where L is constant

From (5) we conclude,

(6) $P(t) = c^{-1} \cdot e^{-c^{-1} \int G(t)dt} \cdot \int e^{c^{-1} \int G(u)du} \cdot F(t) \cdot dt + c_1 \cdot e^{-c^{-1} \int G(t)dt}$,

where $c_1 = \frac{L}{C}$

In case $G(t) = \frac{\alpha}{R}$ where R is the constant resistance then Eq.(6) gets the following form

(7) $P(t) = c^{-1} \cdot e^{-\frac{\alpha(t-t_d)}{c \cdot R}} \cdot \int e^{\frac{\alpha(t-t_d)}{c \cdot R}} \cdot F(t) \cdot dt + c_1 \cdot e^{-\frac{\alpha(t-t_d)}{c \cdot R}}$

In case $G(t) = \pm \frac{\alpha}{R_0}(t-t_d)^n$ for n=1 , Eq. 6 would be developed into

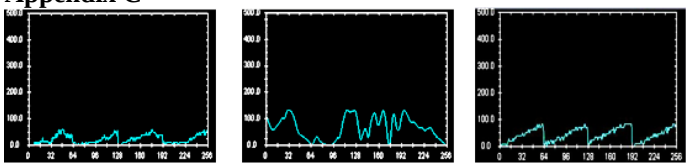
(8) $P(t) = c_1 e^{-\frac{\alpha}{2R_0}(t-t_d)^2 - k} \int e^{\frac{\alpha}{2R_0}(t-t_d)^2 + k} F(t)dt + c_2 e^{-\frac{\alpha}{2R_0}(t-t_d)^2 - k}$

$$= c_1 e^{-\frac{\alpha}{2R_0}(t-t_d)^2} \int e^{\frac{\alpha}{2R_0}(t-t_d)^2} F(t)dt + c_2 e^{-\frac{\alpha}{2R_0}(t-t_d)^2 - k}$$

$$= c_1 e^{-\frac{\alpha}{2R_0}(t-t_d)^2} \int e^{\frac{\alpha}{2R_0}(t-t_d)^2} F(t)dt + c_2 e^{-\frac{\alpha}{2R_0}(t-t_d)^2}$$

while $c_2 = c_1 e^{-k}$.

Appendix C



Three different types of noisy pressure waveform P(t)