Appendix

The selection of the function τ (t):
Table A1 summarizes the constraints that a function for the generation time has to comply.

Constraints		Consequences	
$N = N_0 2^{\frac{t}{\tau}(t)}$	$\lim_{t\to\infty} N = N_{\max}$	$\lim_{t\to\infty}(\frac{t}{\tau}) = \frac{1}{b}$	$\lim_{t\to\infty} \tau = bt$
$\lim_{t\to 0} N = N_0$		$\lim_{t\to 0}(\frac{t}{\tau}) = 0$	
$\frac{\dot{N}}{N} = \log 2 \frac{\tau - t\dot{\tau}}{\tau^2}$	$\lim_{t\to 0}(\frac{\dot{N}}{N}) = \log 2 \lim_{t\to 0}(\frac{1}{\tau} - \frac{t\dot{\tau}}{\tau^2}) = 0$	$\lim_{t\to 0}(\frac{1}{\tau}) = 0$	$\lim_{t\to 0} \tau = \infty$
$\lim_{t\to 0}(1 - \mathrm{t}\frac{\dot{\tau}}{\tau}) = \text{constant}$		$\lim_{t\to 0} \tau = \frac{a}{t}$	$\lim_{t\to 0}(1 - \mathrm{t}\frac{\dot{\tau}}{\tau}) = 2$

Table A1: The constraints come from the empirical evidence of the growth trend and by the expression for a duplication process. The consequences are the simplest relationships that match the constraints.

Therefore a simple expression for τ(t) is:

$$\tau(\mathrm{t}) = \frac{a}{t} + bt$$

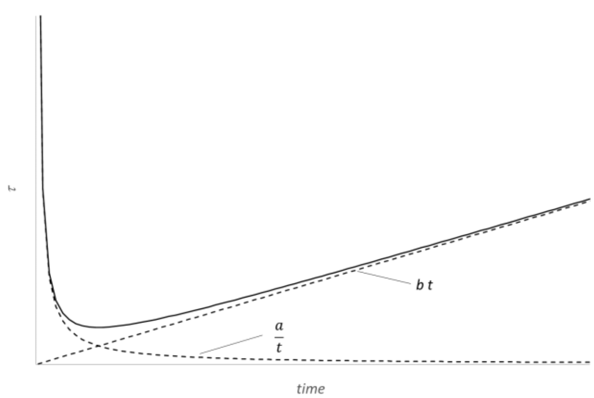

Figure A1: Trend and limit trends of t(t)

Determination of t*:

$$\frac{\dot{N}}{N} = \ln(2)\frac{2at}{(\mathrm{a}+\mathrm{bt}^2)^2} = \frac{d\log N}{dt} \qquad \text{(A1)}$$

$$\frac{d^2 \log N}{dt^2} = \frac{2a\ln(2)}{(\mathrm{a}+\mathrm{bt}^2)^4}[\mathrm{a}^2 - 3\mathrm{b}^2\mathrm{t}^4 - 2\,\mathrm{abt}^2] \qquad \text{(A2)}$$

Putting this expression equal to zero, t^* can be singled out. Replacing t^2 with x and solving the equation $3b^2x^2 + 2abx - a^2 = 0$

One of two roots turns out to be x = a/3b, which means:

$$t^* = \sqrt{\frac{a}{3b}} \qquad \text{(A3)}$$

Accordingly,

$$\tau^* = \sqrt[4]{\frac{ab}{3}} \qquad \text{(A4)}$$

$$\log(\frac{N}{N_0})^* = \log(2)\frac{t^*}{\tau^*} = \frac{\log(2)\sqrt{\frac{a}{3b}}}{\sqrt[4]{\frac{ab}{3}}} = \frac{\log(2)}{4b} = \frac{\log(\frac{N}{N_0})_{\max}}{4} \qquad \text{(A5)}$$

("*" stands for "at $t=t^*$").

Determination of t(0) and tend:

The tangent to the growth curve at t=t* is

$$y = [\frac{1}{8t^*}\log(\frac{N}{N_0})_{\max}](3\mathrm{t} - \mathrm{t}^*) \qquad \text{(A6)}$$

For y=0, the corresponding time is:

$$t(0)=1/3\ t^* \qquad \text{(A7)}$$

For $y=\log(2)/b=\log(N/N_0)_{max}$, one can evaluate the so-called t_{end}:

$$t_{end} = \sqrt{3\frac{a}{b}} = 3t^* \qquad \text{(A8)}$$

The growth curve expressed using the reduced quantity $\xi = b\log_2(\frac{N_{\max}}{N_0}) = \frac{t^2}{(\frac{a}{b}) + \mathrm{t}^2}$ has a tangent at $t=t_{end}$ with

slope

$$\dot{\xi}_{end} = \frac{2kt_{end}}{(k + t_{end}{}^2)^2} \tag{A9}$$

where k=(a/b). Since $t_{end}=3t^*=3(a/3b)^{1/2}=(3k)^{1/2}$, one can easily define the equation of the corresponding straight line,

$$\xi = \frac{1}{8}\sqrt{\frac{3}{k}}t + \delta \tag{A10}$$

Putting the condition of tangency at $t=t_{end}$, namely, $y=\xi(t = t_{end})=0.75$ (equation A8), one gets,

$\xi=3/8$. Looking for the time at which $\xi=1$, one gets $t=5\ (a/3b)=5t^*$, that corresponds at $t_R=5$.

Determination of t_0:

The tangent at t^* intercepts the horizontal axis at $t=t(0)$ and goes through the points (t^*, log N^*/N^0) and (t_{end}, log N_{max}/N_0). When a long flat trend precedes the rising branch of the growth curve, a time shift is necessary to adapt the model to the experimental evidence. Such time shift, t_0, must be subtracted from the observed t^*, t_{end}, and t(0), when applying equations A(3), A(7) and A(8).

The evaluation of t_0 straightforwardly comes from the fact that one simply has to rigidly shift the plot, which means that the slope of the tangent at t^* does not change. To simplify the expressions, it is expedient to represent the growth curve with equation (7), namely, $\xi(t) = \frac{t^2}{k + t^2}$, where k=(a/b). This means that the experimental tangent straight line at $t=t^*$ goes through $\xi^*=0.25$ and has slope $\dot{\xi}^* = \frac{2kt^*}{(k + t^{*2})^2}$ which is the same as in the case with $t_0=0$, namely, $\dot{\xi}^* = \frac{3}{8}\sqrt{\frac{3}{k^{\bullet}}}$. The corresponding straight line equation is:

$$\xi = \dot{\xi}^{*} t - \gamma \tag{A11}$$

where ($-\gamma$) is the intercept at t=0. Now the shift t_0 can be evaluated by imposing $\xi = -\xi^{*}/2 = -0.25$ for $t = t_0$ (see Table 1).

This leads to $$t_0 = \frac{\gamma - 0.125}{3/8\sqrt{3/k}} = -\frac{\text{experimental intercept} + 0.125}{\text{experimental slope}} \tag{A12}$$

or, using the standard quantities of the current practice

$$t_0 = -\frac{\text{experimental intercept} + \log(\frac{N}{N_0})^{*}/2}{\text{experimental slope}} \tag{A13}$$

Equation A(12) can be rewritten taking into account that $\dot{\xi}^{*} = \frac{3}{8}\sqrt{\frac{3}{k}} = \frac{3}{8}\frac{1}{(t^{*})^{id}}$ and that $(t^{*})^{id} = (t^{*} - t_0)$, where $(t^{*})^{id} = (a/3b)^{1/2}$ refers to the case $t_0=0$ (equation A3). This leads to the expression:

$$t_0 = t^{*}\frac{8\gamma - 1}{8\gamma + 2} \tag{A14}$$